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基于刚体运动与弹性运动的机械系统动力学建模研究*

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摘要 讨论了具有刚体运动与柔性变形的机械系统的动力学建模.将刚体自由度与弹性变形自由度看作广义坐标,利用有限元法对具有刚性运动与弹性变形的机械系统的运动与变形进行了描述,得到了以刚体位移与弹性变形位移表示的单元的广义惯性力;从应力应变入手,得到了表示单元弹性变形与几何非线性变形的结构刚度矩阵与几何非线性刚度矩阵,使用Kane方程推导了弹性连杆机构的单元运动方程.这种建模方法,可以使用在任意结构的机械系统.

关键词 柔性变形,有限元法,模态,Kane方程,动力分析

前言

机械系统的动力学分析是建立在连杆刚体假设基础上,应力的产生往往只假定由于表面力和惯性力,由应力来进行机械系统的设计、安装及测试,这是建立在刚体模型之上.当机械系统启动速度很大时,惯性力变得很大,此时连杆件受到较大的变形,在这种情况下,刚体假设也就不再适用了.因此,在进行机械系统的模拟和设计时就必须通过考虑杆子的弹性而精确地模仿机械系统的运动.由于弹性不能够完全忽略,因此必须进行主动控制以便使弹性变形的影响减少到最低,而要如此,就应发展精确而又实际的模仿机械系统的方法.

带弹性体的机械系统的建模,按其建模方式,可分为三方面的工作.

第一种方法[1-5]也是原始的和最简单的方面,把弹性杆件看作是有限个自由变的连续系统,所得的运动方程是非线性偏微方程,这种方法一直被使用来推导运动方程,动力分析及带弹性结构体的动态反应.

第二种方法[6-17]是将弹性体看作是一个具有有限个弹性自由变的离散系统,它使用的是有限元法.使用有限单元法给机械系统建模的一般方法的优点是给机械系统提供了一种系统的建模技术,一般方法及建立一个基础工作.在这些工作中,系统的运动可看作是刚体运动和弹性体运动的叠加.

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第三种方法[18-28]就是使用拉格朗日乘子技术将带有关节的约束并入运动方程.这种方法主要适用于较大规模的动力学系统而不仅仅适用于机械系统.由此导出的公式在计算机上是通用的、简要的和方便的.

此文中,以系统方式,对带弹性体的操作机械手系统进行分析,讨论了具有刚体运动与柔性变形的机械系统的动力学建模.将刚体自由度与弹性变形自由度看作广义坐标,利用有限元法进行运动与变形描述,使用Kane方程推导了弹性连杆机构的运动方程.这种运动方程能够用来分析工业机械操作手.由于使用有限元方法对弹性杆进行建模,因而不管它具有何种复杂的形状.

1 单元的变形描述

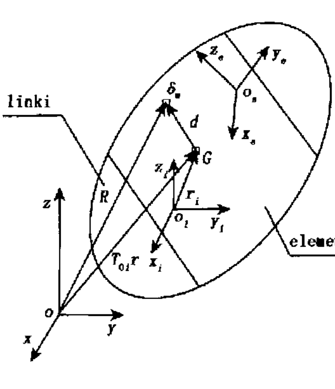

图1 单元的变形描述

Fig.1 Deformation of the element

如图1所示：$oxyz$ 为惯性坐标系，$o_ix_iy_iz_i$ 为连体坐标系，操作机械手系统，其在空间的位移的变化，可用如下关系表示

$$r_i = T_{0i}r（这里\ T_{0i} = T_{01}T_{12}T_{23} \wedge T_{i-1,i}） \tag{1}$$

上式表示从杆1→杆 i 的齐次坐标变换矩阵．一般说来，第 i 个连杆相对于参考坐标系 $oxyz$ 的刚体的平移和旋转变换矩阵 T_{0i} 是一个4×4矩阵，它具有如下形式

$$T_{0i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ x_{ooi} & \cos\theta_{xx_i} & \cos\theta_{xy_i} & \cos\theta_{xz_i} \\ y_{ooi} & \cos\theta_{yx_i} & \cos\theta_{yy_i} & \cos\theta_{yz_i} \\ z_{ooi} & \cos\theta_{zx_i} & \cos\theta_{zy_i} & \cos\theta_{zz_i} \end{bmatrix}$$

上式中的第一列，表示连体坐标系 $o_ix_iy_iz_i$ 的原点相对于惯性参考 $oxyz$ 的坐标；第二、三、四列表示 $x_iy_iz_i$ 轴在参考坐标系 $oxyz$ 中的方向余弦，它由杆件平移和旋转影响，是一个可变的转换矩阵．

如果关节之间只作旋转变形，则

$$T_{i-1,i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_i & -\cos\phi_i & 0 \\ 0 & \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

因而

$$\dot{T}_{i-1,i} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_i & -\sin\phi_i & 0 \\ 0 & \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = Q_iT_{i-1,i}\dot{\phi}_i$$

由此可知

$$\dot{T}_{0i} = Q_1T_{01}\dot{\phi}_1T_{12} \wedge T_{i-1,i} + \wedge + T_{01}T_{02} \wedge Q_{i-1}T_{i-1,i}\dot{\phi}_i = w_i \cdot T_{0i} \tag{2a}$$

这里 $w_i = \sum_{j=1}^{i-1} Q_j\dot{\phi}_j$，称作刚体速度算子矩阵，$Q_j$ 表示常数矩阵．对 $\dot{T}_{0i}$ 求导，可得

$$\ddot{T}_{0i} = \sum_{j=1}^{i-1}(Q_j\ddot{\phi}_j)T_{0i} + \sum_{j=1}^{i-1}(Q_j\dot{\phi}_j)\dot{T}_{0i} = \sum_{j=1}^{i-1}(Q_j\ddot{\phi}_j)T_{0i} + \sum_{j=1}^{i-1}(Q_j\dot{\phi}_j)\sum_{j=1}^{i-1}(Q_k\dot{\phi}_k)T_{0i} \tag{3}$$

这里 $\dot{\phi}_j$ 可以通过如下方式写出：对于一般机械手操作系统，其几何约束方程可写作

$$\phi_i(\varphi_1,\cdots,\varphi_f) = 0 \quad (i = 1,\cdots,N) \tag{4}$$

这里 $\varphi_1,\cdots,\varphi_f$ 表示非完整系统具有的 f 个自由度，i 表示系统物体的个数．由上式求偏导，可定义：$\phi'_{ik} = \delta\phi_i/\delta\varphi_k$．由此可知

$$\dot{\phi}_l = \sum_{k=1}^{f}\phi'_{ik}\dot{\varphi}_k \tag{5}$$

上式可看作是由动力学相容条件得到的杆件的广义速率表达式，故 w_i 可写成如下形式

$$w_i = \sum_{j=1}^{i-1}\sum_{k=1}^{f}Q_j\phi'_{ik}\dot{\varphi}_k = \sum_{k=1}^{f}[\sum_{j=1}^{i-1}Q_j\phi'_{ik}]\dot{\varphi}_k = \sum_{k=1}^{f}\overline{W}_{ik}\dot{\varphi}_k \tag{6}$$

这里，$\overline{W}_{ik} = \sum_{j=1}^{i-1}Q_j\phi'_{jk}$．因而

$$\dot{T}_{0i} = \sum_{j=1}^{i-1}\sum_{k=1}^{f}\phi'_{jk}Q_j\dot{\varphi}_kT_{0i} = \sum_{k=1}^{f}W_{ik}\dot{\varphi}_kT_{0i} \tag{2b}$$

ϕ_i 的二阶导数为

$$\ddot{\phi}_i = \sum_{k=1}^{f}\sum_{l=1}^{f}\frac{\partial^2\phi}{\partial\varphi_k\partial\varphi_l}\dot{\varphi}_k\dot{\varphi}_l + \sum_{k=1}^{f}\phi'_{ik}\ddot{\varphi}_k = \nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l + \sum_{k=1}^{f}\phi'_{ik}\ddot{\varphi}_k \tag{7}$$

故式(3)可表示为

$$\ddot{T}_{0i} = \sum_{j=1}^{i-1}Q_j\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l + \sum_{k=1}^{f}\overline{W}_{ij}\ddot{\varphi}_k)T_{0i} + \overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_kT_{0i} \tag{8}$$

2 单元的运动描述

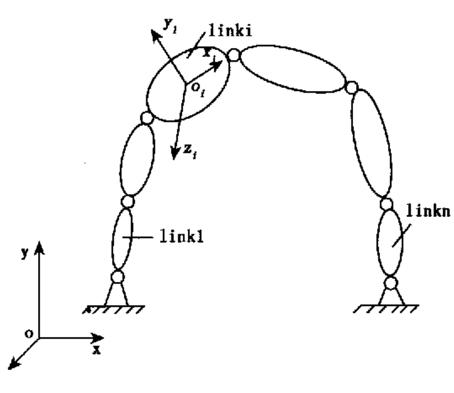

图2 弹性机械系统

Fig. 2 The elastic mechanism system

对于弹性机械系统，如图2所示，取杆件 i 作为研究对象，有限单元 e 的单元质量为 δm，坐标 $oxyz$

为固连于地球的惯性参考坐标系，$o_ix_iy_iz_i$ 是连杆 i 的质心的连体坐标系，坐标 $o_ex_ey_ez_e$ 表示单元坐标系，建在单元 e 的重心上；G 表示有限单元 e 上的微元 δm 在刚体中的位置(如图 1 所示)．

现对单元 e 定义一个附加的 4×4 阶转换矩阵 $\boldsymbol{R}^e$，它是一个相对连杆坐标系 $o_ix_iy_iz_i$ 保持恒向的单元坐标系 $o_ex_ey_ez_e$，故 $\boldsymbol{R}^e$ 是一个常数矩阵，它具有如下形式

$$\boldsymbol{R}^e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos\theta_{x_ix_0} & \cos\theta_{x_iy_e} & \cos\theta_{x_iz_e} \\ 0 & \cos\theta_{y_ix_e} & \cos\theta_{y_iy_e} & \cos\theta_{y_iz_e} \\ 0 & \cos\theta_{z_ix_e} & \cos\theta_{z_iy_e} & \cos\theta_{z_iz_e} \end{bmatrix}$$

转换矩阵 $\boldsymbol{T}_{0i}$ 和 $\boldsymbol{R}^e$ 乘积被用来转化为单元坐标系相对参考坐标系 $oxyz$ 的弹性变形量 $\boldsymbol{d}$．

现在我们来确定单元质点 δm 在参考系中的位置可由向量 $\boldsymbol{R}$ 确定，向量 $\boldsymbol{R}$ 由两部分组成，δm 未变形时(也就是刚体位置)和 δm 弹性变形(如图 1 中所示)．我们将向量 $\boldsymbol{r}$ 表示微元 δm 在杆系坐标中的刚体位置，考虑到微元 δm 在杆件坐标中的刚体位置是不变的，故向量 $\boldsymbol{r}$ 是一个常向量．微元 δm 在参考系中的刚体位置可以用 $\boldsymbol{T}_{0i}\boldsymbol{r}$ 来表示，向量 $\boldsymbol{d}$ 是微元 δm 在有限单元参考系 $x_ey_ez_e$ 中的弹性变形，它在参考坐标系下的弹性变形量可以表示成矩阵乘积 $\boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{d}$．微元 δm 在参考系中的位置，可由下式给出

$$\boldsymbol{R} = \boldsymbol{T}_{0i}\boldsymbol{r} + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{d} \tag{9}$$

根据有限元理论，节点 A(在单元 e 中)在坐标系 $x_ey_ez_e$ 中的弹性变形量 $\boldsymbol{d}$，可以用节点在单元坐标系中的弹性变形向量 $\boldsymbol{u}^e$ 表示，即 δm 的弹性变形向量 $\boldsymbol{d}$ 可以表示成为节点弹性变形向量 $\boldsymbol{u}^e$ 的线性函数

$$\boldsymbol{d} = \boldsymbol{N}^e\boldsymbol{u}^e \tag{10}$$

矩阵 $\boldsymbol{N}^e$ 为与弹性变形 $\boldsymbol{d}$ 相关的节点弹性变形向量 $\boldsymbol{N}^e$ 的有限单元形函数．另一个向量 $\boldsymbol{p}^e$，即单元 e 上节点相对杆系 $x_iy_iz_i$ 的刚体位移，在连杆坐标 $x_iy_iz_i$ 固定情况下，节点的刚体位置是一个常向量，使用这个向量与形函数 $\boldsymbol{N}^e$ 的乘积，单元质点 δm 的刚体位置 $\boldsymbol{r}$ 表示成为

$$\boldsymbol{r} = \boldsymbol{N}^e\boldsymbol{p}^e \tag{11}$$

上述方程适用于等参元的有限单元法，将式(9)和式(10)代入式(8)，则的刚体位置可以表示成为

$$\boldsymbol{R} = \boldsymbol{T}_{0i}\boldsymbol{N}^e\boldsymbol{p}^e + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{u}^e \tag{12}$$

式(12)对时间求导，得质点 δm 在参考坐标系下的速度

$$\dot{\boldsymbol{R}} = \dot{\boldsymbol{T}}_{0i}\boldsymbol{N}^e\boldsymbol{p}^e + \dot{\boldsymbol{T}}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{u}^e + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\dot{\boldsymbol{u}}^e \tag{13a}$$

$\dot{\boldsymbol{T}}_{0i}$ 是 $\boldsymbol{T}_{0i}$ 对时间的导数，$\dot{\boldsymbol{u}}^e$ 是弹性单元体 δm 相对单元坐标系 $x_ey_ez_e$ 的速度向量，形函数矩阵 $\boldsymbol{N}^e$ 和 4×4 阶 $\boldsymbol{R}^e$ 阵是一个常数矩阵，不受对时间的导数的影响，使用方程(2)，上述方程则可以表示成

$$\dot{\boldsymbol{R}} = [\sum_{k=1}^{f}\overline{\boldsymbol{W}}_{ik}\boldsymbol{T}_{0i}(\boldsymbol{N}^e\boldsymbol{p}^e + \boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{u}^e)]\dot{\varphi}_k + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\dot{\boldsymbol{u}}^e \tag{13b}$$

$\boldsymbol{R}$ 的二阶导数可对(13a)求导得到

$$\ddot{\boldsymbol{R}} = \ddot{\boldsymbol{T}}_{0i}\boldsymbol{N}^e\boldsymbol{p}^e + \ddot{\boldsymbol{T}}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{u}^e + 2\dot{\boldsymbol{T}}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\dot{\boldsymbol{u}}^e + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\ddot{\boldsymbol{u}}^e \tag{14}$$

将式(2)与式(8)代入式(14)，可得

$$\ddot{\boldsymbol{R}} = [(\sum_{j=1}^{i-1}\boldsymbol{Q}_j\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l + \sum_{k=1}^{f}\overline{\boldsymbol{W}}_{ij}\ddot{\varphi}_k)\boldsymbol{T}_{0i} + \overline{\boldsymbol{W}}_{ij}\overline{\boldsymbol{W}}_{ik}\dot{\varphi}_j\dot{\varphi}_k\boldsymbol{T}_{0i}]\boldsymbol{N}^e\boldsymbol{p}^e + [(\sum_{j=1}^{i-1}\boldsymbol{Q}_j\nabla^2\phi\dot{\varphi}_k\dot{\varphi}_l + \sum_{k=1}^{f}\overline{\boldsymbol{W}}_{ij}\ddot{\varphi}_k)\boldsymbol{T}_{0i} + \overline{\boldsymbol{W}}_{ij}\overline{\boldsymbol{W}}_{ik}\dot{\varphi}_j\dot{\varphi}_k\boldsymbol{T}_{0i}]\boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{u}^e + 2\sum_{j=1}^{i-1}\overline{\boldsymbol{W}}_{ij}\dot{\varphi}_j\boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\dot{\boldsymbol{u}}^e + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e\ddot{\boldsymbol{u}}^e \tag{15}$$

3　广义偏速度、广义惯性力

以表示刚体位移的自由度 φ_i 与表示表示弹性变形的自由度 u 作为广义坐标，以 $\dot{\varphi}_i$ 与 $\dot{u}$ 作为广义速率，考察式(13b)与式(15)，可以得到单元 δm 的广义偏速度

$$\boldsymbol{v}_1^P = \sum_{k=1}^{f}\overline{\boldsymbol{W}}_{ik}\boldsymbol{T}_{0i}(\boldsymbol{N}^e\boldsymbol{p}^e + \boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{u}^e) + \boldsymbol{T}_{0i}\boldsymbol{R}^e\boldsymbol{N}^e \tag{16}$$

单元 δm 的加速度 $\boldsymbol{a}_0^N = \ddot{\boldsymbol{R}}$，由式(15)给出．相对于第 i 个广义速度的广义惯性力 $F_i^* = \boldsymbol{V}_i^N\boldsymbol{F}^*$，$\boldsymbol{F}^* = -\int_B \rho\boldsymbol{a}_0^N\mathrm{d}x_1\mathrm{d}x_2$，故有广义惯性力

$$F_i^* = -\int_B \rho\boldsymbol{V}_i^P\boldsymbol{a}_0^P\mathrm{d}x_1\mathrm{d}x_2 \tag{17}$$

利用式(15)和式(16)，令

$\boldsymbol{T}_{0i}\boldsymbol{T}_{0i}^{\mathrm{T}}\boldsymbol{N}^e\boldsymbol{N}^{eT}\boldsymbol{p}^e\boldsymbol{p}^{eT} = [\boldsymbol{TNP}]$；

$\boldsymbol{T}_{0i}\boldsymbol{T}_{0i}^{\mathrm{T}}\boldsymbol{R}^e\boldsymbol{N}^e\boldsymbol{N}^{eT}\boldsymbol{p}^e = [\boldsymbol{TrNp}]$；

$\boldsymbol{T}_{0i}\boldsymbol{T}_{0i}^{\mathrm{T}}\boldsymbol{R}^e\boldsymbol{R}^{eT}\boldsymbol{N}^e\boldsymbol{N}^{eT} = [\boldsymbol{TRN}]$，

可将式(17)扩展成如下形式

$$
\begin{aligned}
F_i^* = & -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{i-1}\sum_{l=1}^{l} Q_j\overline{W}_{ik}\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l[TNP]\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{l=1}^{f}\sum_{j=1}^{i-1} Q_j\overline{W}_{ik}\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{f}\overline{W}_{ij}\overline{W}_{ik}\ddot{\varphi}_k[TNP]\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{f}\overline{W}_{ij}\overline{W}_{ik}\ddot{\varphi}_k[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ik}\sum_{j=1}^{f}\overline{W}_{ij}\sum_{k=1}^{f}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_k[TNP]\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ik}\sum_{j=1}^{f}\overline{W}_{ij}\sum_{k=1}^{f}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_k[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{i-1} Q_j\overline{W}_{ik}\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{i-1} Q_j\overline{W}_{ik}\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l[TRN]u^e u^{eT}\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{f}\overline{W}_{ij}\overline{W}_{ik}\varphi_k[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\sum_{k=1}^{f}\overline{W}_{ij}\overline{W}_{ik}\varphi_k[TRN]u^e u^{eT}\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ik}\overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_k[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ik}\overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_k[TRN]u^e u^{eT}\mathrm{d}x_1\mathrm{d}x_2 \\
& -2\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{f}\overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j[TrNp]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -2\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{f}\overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j[TRN]u^e u^{eT}\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ik}[TrNp]\ddot{u}^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ik}[TRN]u^e\ddot{u}^{eT}\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{j=1}^{i-1} Q_j\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l[TrNp]\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{k=1}^{f}\overline{W}_{ij}\ddot{\varphi}_k[TrNp]\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho\overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_k[TrNp]\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{j=1}^{i-1} Q_j\nabla^2\phi_i\dot{\varphi}_k\dot{\varphi}_l[TRN]^{\mathrm{T}}u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{j=1}^{i-1}\overline{W}_{ik}\varphi_k[TRN]u^{e\mathrm{T}}u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho\overline{W}_{ij}\overline{W}_{ik}\dot{\varphi}_j\dot{\varphi}_k[TRN]u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -2\int_B \rho \sum_{j=1}^{i-1}\overline{W}_{ij}\dot{\varphi}_j[TRN]\dot{u}^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho[TRN]\ddot{u}^e\mathrm{d}x_1\mathrm{d}x_2
\end{aligned}
\tag{18}
$$

忽略二阶以上量,保留交叉项,经整理,得:

(1) 以刚体位移 φ 表示的广义惯性力

$$
\begin{aligned}
F_i^* = & -\int_B \rho \sum_{k=1}^{f}\sum_{j=1}^{i-1}\overline{W}_{ij}\{\overline{W}_{ik}([TNP]+ \\
& [TrNp]u^e)+[TrNp]\}\ddot{\varphi}_j\mathrm{d}x_1\mathrm{d}x_2) \\
& -2\int_B \rho \sum_{k=1}^{f}\overline{W}_{ij}\{\sum_{j=1}^{f}\overline{W}_{ik}[TrNp]- \\
& [TRN]\}\dot{u}^e\dot{\varphi}_j\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho \sum_{j=1}^{f}\overline{W}_{ij}\{\sum_{k=1}^{i-1}\overline{W}_{ik}[TrNp]- \\
& [TRN]\}\dot{u}^e\varphi_j\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho\{\sum_{k=1}^{f}\overline{W}_{ik}[TrNp]- \\
& [TRN]\}\ddot{u}^e\varphi_j\mathrm{d}x_1\mathrm{d}x_2
\end{aligned}
\tag{19}
$$

(2) 以弹性位移 u^e 表示的广义惯性力

$$
\begin{aligned}
F_i^* = & -\int_B \rho\{\sum_{k=1}^{f}\overline{W}_{ik}[TrNp]+ \\
& [TRN]\}\ddot{u}^e\varphi_j\mathrm{d}x_1\mathrm{d}x_2 \\
& -2\int_B \rho \sum_{j=1}^{f}\overline{W}_{ij}\{\sum_{k=1}^{f}\overline{W}_{ik}[TrNp]+ \\
& [TRN]\}\dot{\varphi}_j\dot{u}^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho\{\sum_{k=1}^{f}\overline{W}_{ik}\{\sum_{j=1}^{f}\overline{W}_{ij}\ddot{\varphi}_k[TrNp]+ \\
& \sum_{j=1}^{f}\overline{W}_{ij}\varphi_k[TrNp]+\varphi_k[TRN]\}u^e\mathrm{d}x_1\mathrm{d}x_2 \\
& -\int_B \rho\{\sum_{j=1}^{i-1}\overline{W}_{ij}\{\sum_{k=1}^{f}\overline{W}_{ik}[TNP]+ \\
& [TrNp]\}\ddot{\varphi}_k\mathrm{d}x_1\mathrm{d}x_2
\end{aligned}
\tag{20}
$$

4 广义主动力、动力学方程[29]

考虑到 i 物体上任意单元 e 上任一点,其应力-应变关系可表示为

$$
\varepsilon_{\alpha,\beta}=\frac{1}{2}(W_{\alpha,\beta}+W_{\beta,\alpha}+\sum_{r=1}^{3}W_{\gamma,\alpha}W_{\gamma,\beta}) \tag{21}
$$

式中, $W_{\alpha,\beta}=\dfrac{\partial W_\alpha}{\partial x_\beta}$, W_α 为位移分量, x_α 为位置坐标分量.由(21)式可得

$$
\dot{\varepsilon}_{\alpha,\beta}=\frac{1}{2}[\dot{W}_{\alpha,\beta}+\dot{W}_{\beta,\alpha}+\sum_{\gamma=1}^{3}(\dot{W}_{\gamma,\alpha}W_{\gamma,\beta}+W_{\gamma,\alpha}\dot{W}_{\gamma,\beta}] \tag{22}
$$

应力表示为 $\boldsymbol{\sigma} = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]^{T}$. 由此而引起广义主动力可由两部分组成: $F_e = \sum_{j=1}^{n} \boldsymbol{\Phi}^T \boldsymbol{K}_e \boldsymbol{\Phi} u$ (φ 为特征向量阵), $\boldsymbol{K}_e$ 为弹性刚度矩阵, $F_G = \sum_{j=1}^{n} \boldsymbol{\Phi}^T \boldsymbol{K}_G \boldsymbol{\Phi} \boldsymbol{u}$, $\boldsymbol{K}_G$ 为非线性变形刚度矩阵(或称几何非线性刚度阵). 广义主动力 F 可写作 $F_i = F_e + F_G$

由 Kane 方程,可写出动力学方程

$$F_i^* + F_i = 0 \tag{23}$$

这里 F_i^* 为广义惯性力,由式(19) 与式(20) 提供, F 为广义主动力,由式(24) 提供.

5　结束语

对带弹性体的操作机械手系统进行分析,讨论了具有刚体运动与柔性变形的机械系统的动力学建模. 将刚体自由度与弹性变形自由度看作广义坐标,利用有限元法进行对弹性连杆的单元运动与变形进行了描述,并使用 Kane 方程推导了弹性连杆机构的单元运动方程.

弹性连杆机构的系统运动方程,将另行论文讨论.

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DYNAMICS MODELING ANALYSIS OF THE MECHANISM SYSTEM BASED ON RIGID BODY MOTION AND ELASTIC MOTION *

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Abstract The dynamics modeling of the mechanical system with flexible deformation and rigid body motion was discussed. Regarding the rigid body motion degree of freedom and the elastic deformation degree of freedom as the generalized coordinate, and using the finite element method to describe the motion and deformation of the elastic connecting rod with elastic deformation and rigid body motion, we obtained the generalized inertial force in terms of the rigid body displacement and the elastic deformation displacement. Considering the relationship of the stress-strain, we also obtained the structural stiffness matrix, which represents the elastic deformation, and the geometric non-linear stiffness matrix, which represents the nonlinear deformation of the deformed body. Using the Kane equation, we derived the movement equation of the elastic connecting rod organization. This kind of modeling method can be used in the mechanical system with arbitrary structure.

Key words flexible deformation, the finite element method, Kane equation, dynamics analysis

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