巨型混流式水轮机叶片水弹性失稳特性分析

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摘要 巨型混流式水轮机水力振动及其稳定性机理十分复杂,其中由于叶片的水弹性作用诱发的流固耦合振动是重要的影响因素,是导致叶片损失刚度而发生失稳的关键.在正交流线坐标系中,使用基于一般壳体理论及任意拉各朗日-欧拉描述(ALE)建立的叶片流激振动模型,分析了在水弹性作用下,叶片近壁摩擦速度与其动力稳定性的关系,得到了按 Bessel 函数展开的叶片失稳判别条件.

关键词 混流式水轮机,水轮机叶片,水弹性失稳

前言

混流式水轮机组的水力振动机理十分复杂,振动问题随着单机容量、比转速以及转轮尺寸等的增加越来越凸现,已成为威胁机组安全和稳定运行的隐患之一,而当前无论学术界或工程解对诱发振动的机理尚无清楚的理论认识,更没有能有效分析和处理这种复杂振动的理论和方法.叶片作为机组转轮的主要受力和传力部件,直接与携能水流发生相互作用,是机组水力振动的源头所在,因此,从流体力学和振动力学的理论出发,探讨和研究混流式机组水弹性失稳机理及其振动特性显得十分迫切.

流体结构相互作用[1]是诱发水轮机水力振动的主要原因,一方面流道中的流动通过与叶片的作用将其携带的能量传递给叶片而推动转轮的旋转,另一方面叶片在水力的作用下发生振动以及转轮的旋转对流动状态将产生作用,流道内的这种流体和转轮结构之间的相互作用引起水轮机的水力振动,称为流激振动.按当前国际学术界对流激振动较为流行的分类方式[2],流激振动按振因可分为湍流激振、涡激振动、流弹性失稳振动、两相流激振、轴向流诱发振动等5类,对于混流式水轮机,5种激振方式均可能存在,可能是单项激振,也可能是多因素并存激励而诱发振动.

叶片的流弹性失稳诱发的振动是一种与流道流态和叶片翼型紧密相关的参数振动.对于巨型混流式水轮机,由于转轮叶片尺寸大,相对刚度小,在与流体的相互作用过程中,更容易由于流弹性效应导致叶片有效刚度削减而诱发振动.叶片的流弹性失稳振动不同于其他流激振动机理,例如湍流激振和涡激振动[2,3],从机理上讲,水动力和结构弹性变形之间存在相互作用,例如置于静止水中的结构的振动频率明显低于空气中的结构,而在动水中的结构又不同于静水中的结构,结构的有效刚度是流动状态的函数,因此,失稳机理与流态紧密相关.

由于巨型水电站建设的需要,机组单机容量大幅增加,由此带来的振动和稳定性问题十分突出.本文在流道正交流线坐标系下,使用基于一般壳体理论,通过 Navier-Stokes 方程在 ALE 描述下叶片数学模型,分析了叶片水弹性失稳与近壁摩擦速度的关系,得到了叶片动力失稳判别条件及相应的失稳区.

1 叶片流固耦合振动的数学描述

1.1 叶片控制方程

混流式水轮机叶片在如图1所示正交流线坐标系 $\alpha\beta\gamma$ 下,设叶片上任意点 P 沿 α,β 和 γ 方向的位移分量为 u_α,u_β 和 w,按一般薄壳无矩理论[4,5],在忽略 α 和 β 向流体阻尼效应、考虑 γ 向(横向)流体阻尼的情况下,叶片振动微分方程表示为

$$h\rho_s\frac{\partial^2 u_\alpha}{\partial t^2}-\frac{1}{R_\alpha}\frac{\partial N_1}{\partial\theta_\alpha}-\frac{1}{R_\beta}\frac{\partial S}{\partial\theta_\beta}-\tau_{w\alpha}=0 \quad (1)$$

$$h\rho_s\frac{\partial^2 u_\beta}{\partial t^2}-\frac{1}{R_\beta}\frac{\partial N_2}{\partial\theta_\beta}-\frac{1}{R_\alpha}\frac{\partial S}{\partial\theta_\alpha}-\tau_{w\beta}=0 \quad (2)$$

$$h\rho_s\frac{\partial^2 w}{\partial t^2}+\frac{1}{2}\rho_f C_d\left|\frac{\partial w}{\partial t}\right|\frac{\partial w}{\partial t}+\frac{1}{R_\alpha}N_1+\frac{1}{R_\beta}N_2-\bar{p}=0 \quad (3)$$

$$N_1=\frac{Eh(1+\upsilon-\upsilon^2)}{1-\upsilon^2}\left(\frac{1}{R_\alpha}\frac{\partial u_\alpha}{\partial\theta_\alpha}+\frac{w}{R_\alpha}\right)+$$

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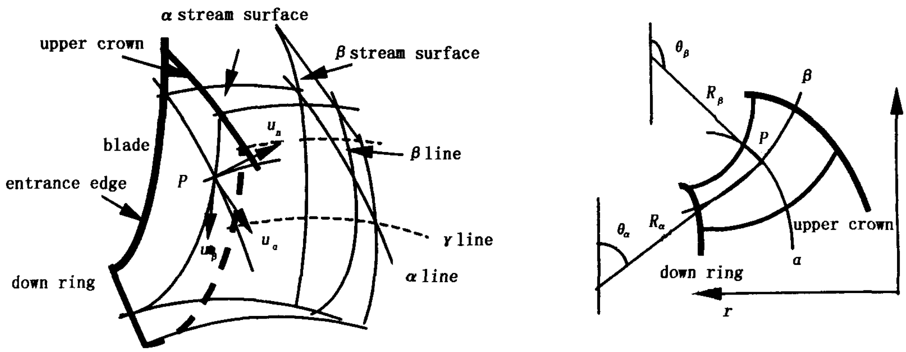

图 1　叶片在正交流线坐标系中的描述

Fig. 1　Blades in the orthogonal streamlines corrdinate system

$$\frac{Eh\upsilon}{1-\upsilon^2}\left(\frac{1}{R_\beta}\frac{\partial u_\beta}{\partial\theta_\beta}+\frac{w}{R_\beta}\right) \tag{4}$$

$$N_2=\frac{Eh}{1-\upsilon^2}\left(\frac{1}{R_\alpha}\frac{\partial u_\alpha}{\partial\theta_\alpha}+\frac{1}{R_\beta}\frac{\partial u_\beta}{\partial\theta_\beta}+\upsilon\frac{w}{R_\alpha}+\frac{w}{R_\beta}\right) \tag{5}$$

$$S=\frac{Eh}{2(1+\upsilon)}\left(\frac{1}{R_\beta}\frac{\partial u_\alpha}{\partial\theta_\beta}+\frac{1}{R_\alpha}\frac{\partial u_\beta}{\partial\theta_\alpha}\right) \tag{6}$$

式中 E 为叶片材料弹性模量;υ 为 Poisson 比;ρ_s 为质量密度;$\tilde{p}$ 为作用在叶片正反表面压力差(或称迫力函数);$\tau_{w\alpha}$ 和 $\tau_{w\beta}$ 为叶片表面粘性摩擦剪应力;N_1 为叶片单位宽度截面上沿 α 方向的张力,N_2 为沿 β 方向的张力,S 为单位宽度中面内的剪力;h 为叶片的厚度;R_α 为 α 线的曲率半径,R_β 为 β 线的曲率半径,C_d 是流体阻力系数.

将 N_1,N_2 和 S 代入式(1) 至式(3),即可得在正交流线坐标系中描述混流式水轮机叶片流激振动的基本方程为

$$\frac{\partial^2 u_\alpha}{\partial t^2}+a_1\frac{\partial u_\alpha}{\partial\theta_\alpha}+a_2\frac{\partial u_\alpha}{\partial\theta_\beta}-a_3\frac{\partial^2 u_\alpha}{\partial\theta_\alpha^2}-a_4\frac{\partial^2 u_\alpha}{\partial\theta_\beta^2}+a_1 w-a_3\frac{\partial w}{\partial\theta_\alpha}-a_5\frac{\partial^2 u_\beta}{\partial\theta_\alpha\partial\theta_\beta}=\frac{\tau_{w\alpha}}{h\rho_s} \tag{7}$$

$$\frac{\partial^2 u_\beta}{\partial t^2}+b_1\frac{\partial u_\beta}{\partial\theta_\beta}+b_2\frac{\partial u_\beta}{\partial\theta_\alpha}-b_3\frac{\partial^2 u_\beta}{\partial\theta_\beta^2}-b_4\frac{\partial^2 u_\beta}{\partial\theta_\alpha^2}-b_5\frac{\partial w}{\partial\theta_\beta}+b_6 w-a_5\frac{\partial^2 u_\alpha}{\partial\theta_\alpha\partial\theta_\beta}=\frac{\tau_{w\beta}}{h\rho_s} \tag{8}$$

$$\frac{\partial^2 w}{\partial t^2}+\frac{m_p}{2h}C_d\left|\frac{\partial w}{\partial t}\right|\frac{\partial w}{\partial t}+c_1\frac{\partial u_\alpha}{\partial\theta_\alpha}+c_2\frac{\partial u_\beta}{\partial\theta_\beta}+c_3 w-\frac{\tilde{p}}{h\rho_s}=0 \tag{9}$$

其中系数为

$$a_1=\frac{E(1+\upsilon-\upsilon^2)\partial R_\alpha}{\rho_s(1-\upsilon^2)R_\alpha^3\partial\theta_\alpha},$$

$$a_2=\frac{E}{2\rho_s R_\beta^3(1+\upsilon)}\frac{\partial R_\beta}{\partial\theta_\beta},$$

$$a_3=\frac{E(1+\upsilon-\upsilon^2)}{\rho_s(1-\upsilon^2)R_\alpha^2}+\frac{E\upsilon}{\rho_s(1-\upsilon^2)R_\alpha R_\beta},$$

$$a_4=\frac{E}{2\rho_s(1+\upsilon)R_\beta^2},$$

$$a_5=\frac{E}{2\rho_s(1-\upsilon)R_\beta R_\alpha},$$

$$b_1=\frac{E}{\rho_s(1-\upsilon^2)R_\beta^3}\frac{\partial R_\beta}{\partial\theta_\beta},$$

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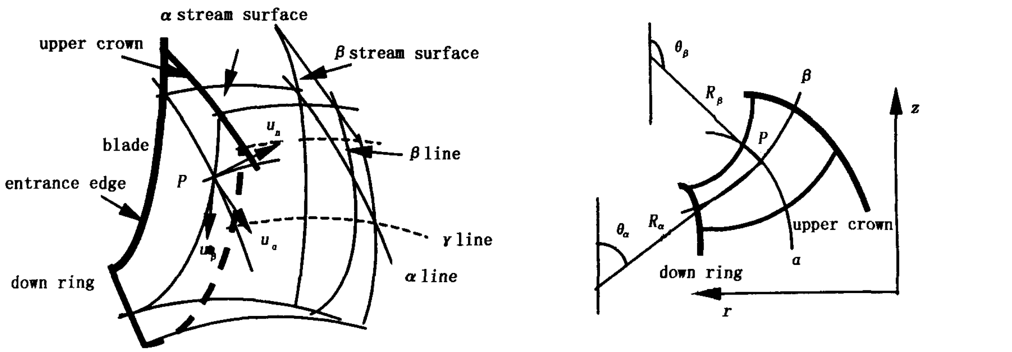
$$c_3=\frac{E}{\rho_s(1-\upsilon^2)R_\alpha R_\beta}\left(2\upsilon+\frac{R_\beta}{R_\alpha}(1+\upsilon-\upsilon^2)+\frac{R_\alpha}{R_\beta}\right).$$

1.2　近壁区流体控制方程

取流道进口的平均径向流速 v 为特征速度,叶



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片进水边与下环交处的半径为特征长度 L，特征时间 $T = L/v$，流动近壁区的厚度为 δ，且认为 $\delta/L \leqslant 1$. 外区域的流动Reynolds数定义为 $R_l = vL/v$，根据文献[6,7]，壁面摩擦速度与流动速度的量级关系取为 $v_w \sim v/\sqrt{R_l}$，近壁区内Reynolds数与流动Reynolds数的量级关系为 $R_w \sim \sqrt{R_l}$，因此，受壁面非滑移条件的影响，近壁区流动的Reynolds数一般不会高，属于低Reynolds数流动. 在近壁区内，设各速度分量的量级关系为：$v_n \sim v_\varphi \sim \dot{u}_r \sim \dot{u}_z \sim \dot{u}_\varphi \sim \varepsilon v_r \sim \varepsilon v_z \sim \varepsilon v$，其中 $\varepsilon = \delta/L$，并对于柱坐标系中在ALE框架下[8~9]描述流道内流体运动的Navier-Stokes方程各相关项做量级分析（在近壁区内），得叶片近壁区流体在ALE描述下的运动方程为

$$\frac{\partial v_r}{\partial t} + \frac{\mathrm{d} v_r}{\mathrm{d} t} + \frac{1}{\rho_f}\frac{\partial p}{\partial r} - \frac{\mu}{\rho_f}\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi^2} - r\tilde{\omega}^2 = 0 \tag{10}$$

$$\frac{\partial v_z}{\partial t} + \frac{\mathrm{d} v_z}{\mathrm{d} t} + \frac{1}{\rho_f}\frac{\partial p}{\partial z} - \frac{\mu}{\rho_f}\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \varphi^2} = 0 \tag{11}$$

$$\frac{1}{\rho_f}\frac{1}{r}\frac{\partial p}{\partial \varphi} - \frac{\mu}{\rho_f}\frac{2}{r^2}\frac{\partial^2 v_r}{\partial \varphi} + 2 v_r \tilde{\omega} = 0 \tag{12}$$

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0 \tag{13}$$

式中 $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}t} + \frac{\partial}{\partial \varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}t} + \frac{\partial}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t} = v_r\frac{\partial}{\partial_r} + \frac{v_\varphi - \dot{u}_\varphi}{r}\frac{\partial}{\partial \varphi} + v_z\frac{\partial}{\partial z}$，$\tilde{\omega}$ 为转轮旋转角速度，r 为计算点到转轮轴线的旋转半径，μ 为流体的动力粘性系数. 在叶片壁面上（$\delta = 0$），流体质点粘附在叶片上，界面条件 $v_r = \dot{u}_r$，$v_\varphi = \dot{u}_\varphi$，$v_z = \dot{u}_z$. 这样，壁面上的流体质点在ALE描述下的应满足的物面方程为

$$\frac{\partial v_r}{\partial t} = -\frac{1}{\rho_f}\frac{\partial p}{\partial r} + \frac{\mu}{\rho_f}\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi^2} + \tilde{\omega}^2 r \tag{14}$$

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho_f}\frac{\partial p}{\partial z} + \frac{\mu}{\rho_f}\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \varphi^2} \tag{15}$$

在式(7)至式(9)中，叶片运动方程涉及到4类与流动状态有关的流体变量. 第1类、第2类流体变量分别为壁面粘性摩擦剪应力 $\tau_{w\alpha}$ 和 $\tau_{w\beta}$ 以及作用在叶片表面的迫力函数 p，由于内区Reynolds数一般不高，可用DNS[10]计算内区流场而定；第3类流体变量为外区域的流速场，可按势流理论分析[11]，也可考虑内、外区的相互影响按内外区耦合分析[12]；第4类流体变量为流线的曲率半径 R_α. 第1类、第2类和第3类变量涉及十分复杂的CFD计算问题，已超出本文讨论的范畴，此处只讨论流线的曲率半径 R_α 的求法.

如图2所示可知，$\theta_\alpha = 90 - \lambda$，$\theta_\beta = 180 - \lambda$，$\theta_\beta = 90 + \theta_\alpha$，在轴面投影面内分别对径向和轴向流速分量求时间的导数，有

$$\frac{\partial v_r}{\partial t} = \frac{\partial}{\partial t}(v_m\cos\theta_\alpha) = -v_m\sin\theta_\alpha\frac{\mathrm{d}\theta_\alpha}{\mathrm{d}t} + \cos\theta_\alpha\frac{\partial v_m}{\partial t} = -v_m\cos\lambda\frac{\mathrm{d}\theta_\alpha}{\mathrm{d}t} + \sin\lambda\frac{\partial v_m}{\partial t} \tag{16}$$

$$\frac{\partial v_z}{\partial t} = \frac{\partial}{\partial t}(v_m\sin\theta_\alpha) = v_m\cos\theta_\alpha\frac{\mathrm{d}\theta_\alpha}{\mathrm{d}t} + \sin\theta_\alpha\frac{\partial v_m}{\partial t} = v_m\sin\lambda\frac{\mathrm{d}\theta_\alpha}{\mathrm{d}t} + \cos\lambda\frac{\partial v_m}{\partial t} \tag{17}$$

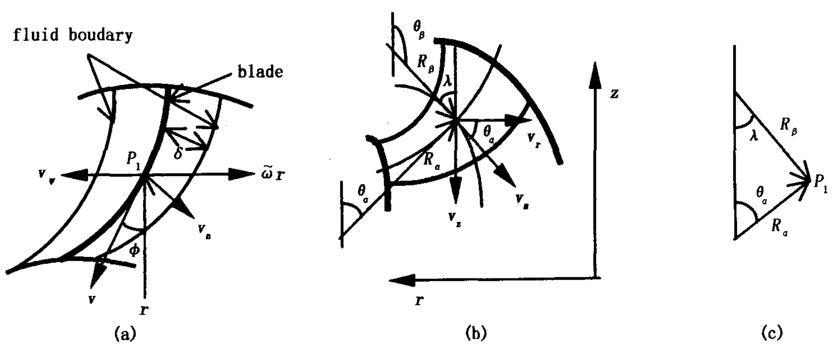

图2　流线曲率半径与流速分量的几何关系

Fig. 2　Near-walled regions, curvature radius and velocity components

又注意到$\frac{d\theta_\alpha}{dt}=\frac{d\theta_\alpha}{d\alpha}\frac{d\alpha}{dt}=\frac{1}{R}v_m$,$\frac{\partial v_m}{\partial t}=\frac{\partial v_m}{\partial \alpha}\frac{d\alpha}{dt}=v_m\frac{\partial v_m}{\partial \alpha}$,将其分别代入式(16)和式(17)得

$$\frac{\partial v_r}{\partial t}=-v_m^2\frac{\cos\lambda}{R_\alpha}+v_m\sin\lambda\frac{\partial v_m}{\partial\alpha} \tag{18}$$

$$\frac{\partial v_z}{\partial t}=v_m^2\frac{\sin\lambda}{R_\alpha}+v_m\cos\lambda\frac{\partial v_m}{\partial\alpha} \tag{19}$$

将式(18)和式(19)代入式(14)和式(15),得关于$1/R_\alpha$和$\partial v_m/\partial\alpha$的方程组为

$$v_m^2\cos\lambda\frac{1}{R_\alpha}-v_m\sin\lambda\frac{\partial v_m}{\partial\alpha}=\frac{1}{\rho_f}\frac{\partial p}{\partial r}-\frac{\mu}{\rho_f}\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}(v_m\sin\lambda)-\bar{\omega}^2 r \tag{20}$$

$$v_m^2\sin\lambda\frac{1}{R_\alpha}+v_m\cos\lambda\frac{\partial v_m}{\partial\alpha}=-\frac{1}{\rho_f}\frac{\partial p}{\partial z}+\frac{\mu}{\rho_f}\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}(v_m\cos\lambda) \tag{21}$$

对$1/R_\alpha$和$\partial v_m/\partial\alpha$求解即可得$\alpha$线的曲率半径为

$$\frac{1}{R_\alpha}=\frac{1}{v_m^2\rho_f}\left(\cos\lambda\frac{\partial p}{\partial r}-\sin\lambda\frac{\partial p}{\partial z}\right)+\frac{\mu}{\rho_f}\frac{1}{v_m^2 r^2}\left[\sin\lambda\frac{\partial^2}{\partial\varphi^2}(v_m\cos\lambda)-\cos\lambda\frac{\partial^2}{\partial\varphi^2}(v_m\sin\lambda)\right]-\frac{\bar{\omega}^2 r}{v_m^2}\cos\lambda \tag{22}$$

对于β线的曲率半径,由图 2 所示的R_α与R_β满足的三角关系可知$\tan\lambda=R_\alpha/R_\beta$.近壁区流体粘性效应不能忽视,但这里出于简化的考虑,暂且忽略流体粘性的影响,则曲率半径及速度沿α线的梯度为

$$\frac{1}{R_\alpha}=\frac{1}{v_m^2}\frac{1}{\rho_f}\left(\cos\lambda\frac{\partial p}{\partial r}-\sin\lambda\frac{\partial p}{\partial z}\right)-\frac{\bar{\omega}^2 r}{v_m^2}\cos\lambda \tag{23}$$

$$\frac{\partial v_m}{\partial\alpha}=-\frac{1}{v_m}\frac{1}{\rho_f}\left(\sin\lambda\frac{\partial p}{\partial r}+\cos\lambda\frac{\partial p}{\partial z}\right)+\frac{\bar{\omega}^2 r}{v_m}\sin\lambda \tag{24}$$

由此可见,流线曲率半径是相对流速以及压力梯度和角坐标的函数,只要流场确定,即可利用所建立的叶片运动微分方程分析叶片的振动,或按流体结构相互作用的方法直接求解流场和叶片的振动.

2　叶片动力稳定性分析

设叶片任意阶正规模态振动为$u_{\alpha n}=U_{\alpha n}\cos\omega_n t$,$u_{\beta n}=U_{\beta n}\cos\omega_n t$,$w_n=W_n\cos\omega_n t$,使用文献[13]模态线的概念(可用于线性和非线性系统),叶面内的振动可以表示为

$$u_{\alpha n}=\varepsilon_{\alpha n}w_n,\ u_{\beta n}=\varepsilon_{\beta n}w_n \tag{25}$$

式中$\varepsilon_{\alpha n}=U_{\alpha n}/W_n$,$\varepsilon_{\beta n}=U_{\beta n}/W_n$,$U_{\alpha n}$,$U_{\beta n}$,$W_n$为第$n$阶模态振动的幅值,$\omega_n$为叶片振动的角频率.参照文献[14]柱壳的结果,将叶片迫力函数$\tilde{p}$可近似表示为

$$\tilde{p}=\tilde{p}_+-\tilde{p}_-=-\eta\rho_f R_\alpha\frac{\partial^2 w}{\partial t^2} \tag{26}$$

式中η为系数,与叶片正(+)和反(-)两面近壁区的流速分布有关.这样,流固耦合情况下叶片横向振动方程可表示为

$$(h\rho_s+\eta\rho_f R_\alpha)\frac{\partial^2 w}{\partial t^2}+\frac{1}{2}\rho_f C_d\left|\frac{\partial w}{\partial t}\right|\frac{\partial w}{\partial t}+c_1\varepsilon_\alpha\frac{\partial w}{\partial\theta_\alpha}+c_2\varepsilon_\beta\frac{\partial w}{\partial\theta_\beta}+c_3 w=0 \tag{27}$$

若将$\theta_\alpha=90-\lambda$和$\theta_\beta=180-\lambda$代入,将方程用角坐标λ表示,则

$$(h\rho_s+\eta\rho_f R_\alpha)\frac{\partial^2 w}{\partial t^2}+\frac{1}{2}\rho_f C_d\left|\frac{\partial w}{\partial t}\right|\frac{\partial w}{\partial t}-(c_1\varepsilon_\alpha+c_2\varepsilon_\beta)\frac{\partial w}{\partial\lambda}+c_3 w=0 \tag{28}$$

其中系数为

$$c_1=\frac{\hat{E}h}{R_\alpha^2}(1+\upsilon-\upsilon^2+\tan\lambda),$$

$$c_2=\frac{\hat{E}h}{R_\alpha^2}(\upsilon+\tan\lambda)\tan\lambda,$$

$$c_3=\frac{\hat{E}h}{R_\alpha^2}(1+\upsilon-\upsilon^2+2\upsilon\tan\lambda+\tan^2\lambda),$$

为便于分析,引入两个无量纲数,$\bar{w}=w/D$,$\bar{t}=t/T$,其中D为转轮直径,T为流体在流道中流动的特征时间,代入式(28)得如下无量纲形式的控制方程

$$\bar{m}_0\frac{\partial^2\bar{w}}{\partial\bar{t}^2}+\bar{c}\left|\frac{\partial\bar{w}}{\partial\bar{t}}\right|\frac{\partial\bar{w}}{\partial\bar{t}}-(\bar{c}_1\varepsilon_\alpha+\bar{c}_2\varepsilon_\beta)\frac{\partial\bar{w}}{\partial\lambda}+\bar{c}_3\bar{w}=0 \tag{29}$$

$$\bar{c}_1=\frac{\hat{E}T^2}{R_\alpha^2\rho_s}(1+\upsilon-\upsilon^2+\tan\lambda),$$

$$\bar{c}_2=\frac{\hat{E}T^2}{R_\alpha^2\rho_s}(\upsilon+\tan\lambda)\tan\lambda,$$

$$\bar{c}_3=\frac{\hat{E}T^2}{R_\alpha^2\rho_s}(1+\upsilon-\upsilon^2+2\upsilon\tan\lambda+\tan^2\lambda),$$

$$m_0=1+\eta\frac{\rho_f}{\rho_s}\frac{R_\alpha}{h},$$

$$\bar{c} = \frac{1}{2}\frac{\rho_f}{\rho_s}\frac{D}{h}C_d,$$

$$\hat{E} = \frac{E}{1-\upsilon^2}$$

若将叶片横向振动按 Bessel 函数展开,并将第 n 阶模态运动表示为 $\bar{w}_n = q_n(\bar{t})W_n(\lambda)$,其中 $W_n(\lambda) = J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)$,$\hat{\beta}_{0,n}$ 为第 1 类零阶 Bessel 函数 J_0 的零点,叶片横向振动方程式(28) 变为

$$\bar{m}_0\ddot{q}_n(\bar{t}) + \bar{c}\,|\dot{q}_n(\bar{t})|\,\dot{q}_n(\bar{t})\,|W_n(\lambda)| - (\bar{c}_1\varepsilon_{\alpha n} + \bar{c}_2\varepsilon_{\beta n})q_n(\bar{t})\frac{W_n'(\lambda)}{W_n(\lambda)} + \bar{c}_3 q_n(\bar{t}) = 0 \tag{30}$$

注意到

$$W_n'(\lambda) = J_0'(\hat{\beta}_{0,n}\lambda/\lambda_0) = -\frac{\hat{\beta}_{0,n}}{\lambda_0}J_1(\hat{\beta}_{0,n}\lambda/\lambda_0),$$

式(30) 成为

$$\ddot{q}_n(\bar{t}) + \frac{\bar{c}}{\bar{m}_0}|J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)|\,|\dot{q}_n(\bar{t})|\,\dot{q}_n(\bar{t}) + \frac{1}{\bar{m}_0}\left[\bar{c}_3 + (\bar{c}_1\varepsilon_{\alpha n} + \bar{c}_2\varepsilon_{\beta n})\frac{\hat{\beta}_{0,n}}{\lambda_0}\times\frac{J_1(\hat{\beta}_{0,n}\lambda/\lambda_0)}{J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)}\right]q_n(\bar{t}) = 0 \tag{31}$$

忽略流体阻尼项后,振动方程式(29) 变为变系数线性微分方程.为不失一般性,设式(25) 中系数 $\varepsilon_{\alpha n}$ 和 $\varepsilon_{\beta n}$ 是时间的某一函数,例如取为

$$\varepsilon_{\alpha n} = \varepsilon_{\alpha 0}\cos\sqrt{\frac{\bar{c}_3}{\bar{m}_0}}\bar{t} = \varepsilon_{\alpha 0}\cos\bar{\omega}_0\bar{t},$$

$$\varepsilon_{\alpha 0} = \max[\varepsilon_{\alpha n}],\ n = 1,2,\cdots,N$$

$$\varepsilon_{\beta n} = \varepsilon_{\beta 0}\cos\sqrt{\frac{\bar{c}_3}{\bar{m}_0}}\bar{t} = \varepsilon_{\beta 0}\cos\bar{\omega}_0\bar{t},$$

$$\varepsilon_{\beta 0} = \max[\varepsilon_{\beta n}],\ n = 1,2,\cdots,N$$

则,式(31) 为

$$\ddot{q}_n(\bar{t}) + b_n|\dot{q}_n(\bar{t})|\dot{q}_n(\bar{t}) + (c_n + \varepsilon_n\cos\bar{\omega}_0\bar{t})q_n(\bar{t}) = 0 \tag{32}$$

式中

$$b_n = \frac{\bar{c}}{\bar{m}_0}|J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)|,\ c_n = \frac{\bar{c}_3}{\bar{m}_0},$$

$$\varepsilon_n = \frac{\bar{c}_1\varepsilon_{\alpha 0} + \bar{c}_2\varepsilon_{\beta 0}}{\bar{m}_0}\frac{\hat{\beta}_{0,n}}{\lambda_0}\frac{J_1(\hat{\beta}_{0,n}\lambda/\lambda_0)}{J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)}$$

显然为典型的 Mathieu 方程,其振动幅值用 Bogoliubov-Mitropolsky 法求得[15]

$$A_n = \frac{3\pi}{4b_n}\sqrt{\varepsilon_n^2 - 4c_n^2} \tag{33}$$

将各参数代入得

$$A_n = \frac{3\pi\bar{m}_0}{4\bar{c}\,|J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)|}\times\sqrt{\frac{(\bar{c}_1\varepsilon_{\alpha 0} + \bar{c}_2\varepsilon_{\beta 0})^2}{\bar{m}_0^2}\left[\frac{\hat{\beta}_{0,n}}{\lambda_0}\frac{J_1(\hat{\beta}_{0,n}\lambda/\lambda_0)}{J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)}\right]^2 - 4\left(\frac{\bar{c}_3}{\bar{m}_0}\right)^2} \tag{34}$$

进而可讨论系统的稳定性问题.作为特例,当忽略流体阻尼项时,系统退化为变系数线性系统,在 1/4 区域失稳的判别条件为

$$\frac{1}{4} - \frac{1}{2}|\varepsilon_n| < c_n < \frac{1}{4} + \frac{1}{2}|\varepsilon_n| \tag{35}$$

将相关参数代入即得叶片失稳区

$$\frac{\bar{m}_0}{4} - \frac{(\bar{c}_1\varepsilon_{\alpha 0} + \bar{c}_2\varepsilon_{\beta 0})}{2}\frac{\hat{\beta}_{0,n}}{\lambda_0}\left|\frac{J_1(\hat{\beta}_{0,n}\lambda/\lambda_0)}{J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)}\right| < \bar{c}_3 < \frac{\bar{m}_0}{4} + \frac{(\bar{c}_1\varepsilon_{\alpha 0} + \bar{c}_2\varepsilon_{\beta 0})}{2}\times\frac{\hat{\beta}_{0,n}}{\lambda_0}\left|\frac{J_1(\hat{\beta}_{0,n}\lambda/\lambda_0)}{J_0(\hat{\beta}_{0,n}\lambda/\lambda_0)}\right| \tag{36}$$

3 算例

某混流式水轮机组转轮直径 6.5 m,叶片翼型如图 3 所示,定性分析叶片出水边的动力稳定性问题.分析中取基本参数为:转速 120 r/min,叶片与上冠交线出水边处 $\lambda_{u0} = 0.50$ rad,与下环交线出水边处 $\lambda_{d0} = 0.0$ rad,$\lambda_0 = 0.50$ rad;出水边厚度 $h = 0.02$ m;材料弹性模量 $E = 206$ GPa,Poisson 比 $\upsilon = 0.3$. 近似取叶片出水边附近壁面压力梯度 $\partial p/\partial r = 0.71$ MPa/m,$\partial p/\partial z = 0$,近壁区摩擦速度按 0.05 倍流动速度考虑,$\varepsilon_{\alpha n} = \varepsilon_{\beta n} = \varepsilon_0 = 0.5$. 线性系统对应 $r = 4.5$ m部位的前4阶模态振动的失稳区如图 4 所示.

4 结语

混流式水轮机组的水力振动及其稳定性问题一直未得到很好的解决,机组振动、转轮叶片出现裂纹,甚至断裂的事例屡见不鲜,是困扰国际学术界和工程界的学术和技术难题.本文在建立描述叶片流激振动控制方程的基础上,利用非线性参数振动的相关理论和方法,研究了叶片横向振动稳定性及其分析方法,从一般的理论角度探索了机组叶片的水力振动机理.

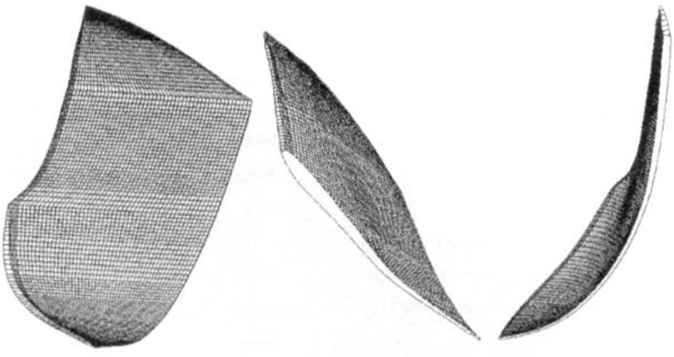

图 3　算例机组叶片翼型

Fig. 3　Blade and its airfoil in working example

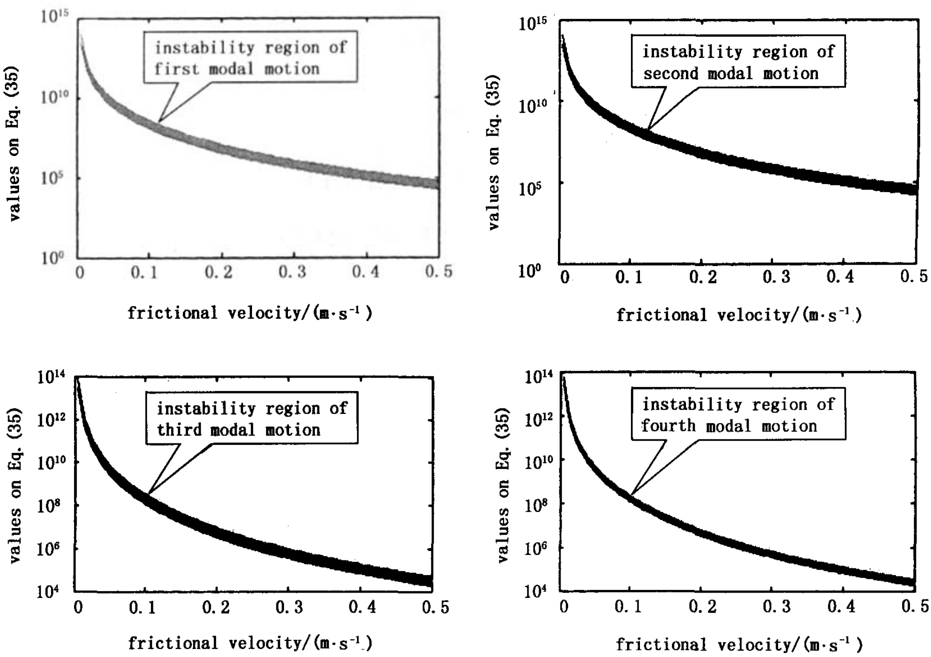

图 4　前 4 阶模态运动随摩擦速度的失稳区

Fig. 4　Instability regions of first 4 modal motions with frictional velocity

参 考 文 献

1 张立翔,杨柯. 流体结构互动理论及其应用. 北京: 科学出版社, 2004(Zhang Lixiang, Yang Ke. The theory of fluid-structure interaction and its applications. Beijing: Science Press, 2004(in Chinese))

2 Weaver DS, Ziada S, Au-Yang MK, Chen SS, Paidoussis MP, Pettigrew MJ. Flow-induced vibration in power and process plants components-progress and prospects. *ASME Journal of Pressure Vessel Technology*, 2000, 122: 339～348

3 Blevins RD. Fluid induced vibrations. 2nd Edition. New York: van Nostrand Reinhold, 1990

4 韩强,黄小清,宁建国. 高等板壳理论. 北京: 科学出版社, 2002(Han Qiang, Huang Xiaoqing, Ning Jianguo. Advanced plate and shell theories. Beijing: Science Press, 2002(in Chinese))

5 成祥生. 应用板壳理论. 济南:山东科学技术出版社, 1989(Cheng Qiangsheng. Applied plate and shell theories. Jinan: Shandong Science and Technology Press, 1989(in Chinese))

6 Piomelli U. Large-eddy and direct simulation of turbulent flows, a CFD Odyssey. Report of Department of Mechani-