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几何非线性的损伤粘弹性 Timoshenko 梁的动力学行为*

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摘要　从考虑损伤的粘弹性材料的一种卷积型本构关系出发，建立了在有限变形下损伤粘弹性 Timoshenko 梁的控制方程.利用 Galerkin 方法对该组方程进行简化，得到一组非线性积分-常微分方程.然后应用非线性动力学数值分析方法，如相平面图，Poincare 截面分析了载荷参数对非线性损伤粘弹性 Timoshenko 梁动力学性能的影响.特别考察了损伤对粘弹性梁的动力学行为的影响.

关键词　损伤粘弹性固体，Timosenko 梁，几何非线性，混沌

引言

粘弹性材料是自然界广泛存在的一类材料，例如聚合物、复合材料、岩石、混凝土等.同时，工程中所用的材料，有的在自然状态下就是一种明显的多孔介质，如混凝土、木材、石料和陶瓷等；有的材料由于冷热加工过程，载荷与温度的变化，化学和射线的作用以及其它多种环境的影响，使材料内部存在和产生微观的以至宏观的缺陷，造成材料力学性能的逐步劣化，从而使结构强度明显削弱，寿命缩短.所以通常材料的劣化-损伤是不可避免的，这早就引起材料科学、力学、工程设计和生产部门工作者的相当重视.

对粘弹性 Euler-Bernoulli 梁和 Timoshenko 梁的静、动力学行为的研究常常采用 Galerkin 方法，其本构关系多为微分型，Boltzmann 叠加原理或 Leaderman 本构关系.1995 年 Suire 和 Cederbaum[1]研究了大变形线性粘弹性梁的周期和混沌动力学行为，其粘弹性本构关系为 Boltzmann 叠加原理。1996 年，Argyris[2]采用微分型本构关系研究了粘弹性梁的混沌运动，2000 年，陈立群和程昌钧[3]采用 Galerkin 方法和微分动力系统的数值方法研究了非线性粘弹性 Euler-Bernoulli 梁的动力学行为，其粘弹性本构关系采用 Leaderman 非线性本构关系.2001 年，李国根和朱正佑等[4,5]利用分数导数型本构关系分析 Timoshenko 梁的动力学行为，对于结构动力学行为，特别是非线性动力学行为，往往采用 Galerkin 截断，本文既考虑材料的粘性，同时又计及材料的损伤，从材料的 Boltzmann 本构定律[6,7]和考虑空洞损伤的线弹性本构规律[8]出发，建立了损伤粘弹性铁木辛柯梁的运动微分方程，并应用 Galerkin 方法和非线性动力学数值分析方法，在数值上分析了损伤粘弹性 Timoshenko 梁丰富的动力学行为.同时，在相同的载荷和材料参数条件下，分析比较了有损伤和无损伤梁的动力学行为.

1　损伤粘弹性运动微分方程

设 u_i，ϵ_{ij}，σ_{ij} 和 $Đ$ 分别是损伤粘弹性材料的位移、应变、应力分量及损伤变量，它们均是坐标 x_i 和时间 t 的函数.根据连续介质力学的基本规律，它们满足如下方程.

运动微分方程

$$\sigma_{ij,j} + f_i - \rho\ddot{u}_i = 0 \tag{1}$$

$$\rho k\ddot{Đ} - \alpha Đ_{,ii} + \omega Đ + \xi(Đ - Đ^0) - \beta\epsilon_{kk} + l = 0 \tag{2}$$

几何方程

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right) \tag{3}$$

本构方程[8]

$$\sigma_{ij} = C_1 \otimes \epsilon_{ij} + C_2 \otimes \epsilon_{kk}\delta_{ij} - \beta(Đ - Đ^0)\delta_{ij} \tag{4}$$

上式中 C_1 和 C_2 为粘弹性材料的性质函数[9]，它们和蠕变函数 J_1 和 J_2 的关系为 $C_1 = L^{-1}[1/(s^2\bar{J}_1)]$，$C_2 = L^{-1}[(\bar{J}_1 - \bar{J}_2)/s^2\bar{J}_1(\bar{J}_1 + 2\bar{J}_2)]$.式中$\overline{(\cdot)}$ 和 L^{-1} 分别表示 Laplace 变换和逆变换，s 是变换参数.符号 $\otimes$ 是 Boltzmann 算子，定

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义为

$$\varphi_1(t)\otimes\varphi_2(t)=\varphi_1(0^+)\varphi_2(t)+\dot{\varphi}_1(t)*\varphi_2(t)=\varphi_1(0^+)\varphi_2(t)+\int_{0^+}^{t}\dot{\varphi}_1(t-\tau)\varphi_2(\tau)\mathrm{d}\tau$$

在(1)～(4)式中,f_i 为已知体积力,ρ 为参考构形的已知密度,k 为已知平衡惯量,l 是已知外在平衡体积力. α,ω,ξ,β 为材料的特征常数,$Đ^0$ 是初始损伤.

若材料的泊松比 $\mu(t)\equiv\mu=$ 常数,则有

$$J_2(t)/J_1(t)=(1-2\mu)/(1+\mu),$$
$$C_2(t)/C_1(t)=\mu/(1-\mu)=\mu_1$$

2 损伤粘弹性 Timoshenko 梁的运动微分方程

Euler-Bernoulli 梁的理论是建立在平截面假设的基础之上,该理论对于细长梁能够给出较为精确的结果.但对于短而粗的梁,这种理论就不再适用.二十世纪20年代,Timoshenko 提出了梁的修正理论,保留了平截面的假设,但认为梁的横截面变形后发生了一个转角. Timoshenko 梁理论计及了横截面剪切变形和转动惯性的影响.

设 ox 轴为截面的中性轴,oy,oz 轴为截面的惯性主轴.设梁是等截面的,横截面面积为 A,高为 h,长为 L,密度为 ρ.若载荷作用于梁的载荷 $q(x,t)$ 作用在 xy 平面内,则认为梁处于平面弯曲状态. 根据 Timoshenko 梁理论[10]. 位移可设为

$$\begin{cases}u_1(x,y,t)=u(x,t)+y\varphi(x,t)\\ u_2(x,y,t)=v(x,t)\end{cases}\tag{5}$$

式中 φ 表示 y 轴的转角. 设梁不受轴力作用,有 $u(x,t)=0$.根据有限变形理论,由位移可得

$$\begin{cases}\varepsilon_x=y\dfrac{\partial\varphi}{\partial x}+\dfrac{1}{2}y^2(\dfrac{\partial\varphi}{\partial x})^2+\dfrac{1}{2}(\dfrac{\partial v}{\partial x})^2\\ \gamma_{xy}=\dfrac{\partial v}{\partial x}+\varphi+y\varphi\dfrac{\partial\varphi}{\partial x}\\ \varepsilon_y=\varepsilon_z=\gamma_{yz}=\gamma_{xz}=0\end{cases}\tag{6}$$

根据梁的假设,应力分量 $\sigma_y,\sigma_z,\tau_{yz}$ 和 τ_{xz} 的影响可忽略. 由(4)和(6)式可得

$$\begin{cases}\sigma_x=(C_1-C_2)\otimes[y\dfrac{\partial\varphi}{\partial x}+\dfrac{1}{2}y^2(\dfrac{\partial\varphi}{\partial x})^2+\dfrac{1}{2}(\dfrac{\partial v}{\partial x})^2]-\beta\hat{Đ}\\ \tau_{xy}=\dfrac{1}{2}C_1\otimes(\dfrac{\partial v}{\partial x}+\varphi+y\varphi\dfrac{\partial\varphi}{\partial x})\end{cases}\tag{7}$$

其中损伤增量 $\hat{Đ}=Đ-Đ^0$. 根据 Cowin 的理论[8],损伤增量可假设为 y 坐标的三次函数,即 $\hat{Đ}(x,y,t)=\hat{D}(x,t)(\frac{y^3}{3}-\frac{h^2}{4}y)$,因而,损伤增量 $\hat{Đ}(x,y,t)$ 满足 $y=\pm\frac{h}{2}$ 表面上,$\frac{\partial\hat{Đ}(x,y,t)}{\partial y}=0$ 的条件. 在有限变形情况下,梁的平衡方程为[4]

$$\begin{cases}\dfrac{\partial}{\partial x}(T_x\dfrac{\partial v}{\partial x})+\dfrac{\partial Q_y}{\partial x}+q(x,t)=\rho A\dfrac{\partial^2 v}{\partial t^2}\\ \dfrac{\partial}{\partial x}(M_z-P_z\dfrac{\partial\varphi}{\partial x})+R_y\dfrac{\partial\varphi}{\partial x}-\dfrac{\partial}{\partial x}(R_y\varphi)-Q_y=\rho I_z\dfrac{\partial^2\varphi}{\partial t^2}\end{cases}\tag{8}$$

其中 $T_x=\iint_A\sigma_x\mathrm{d}A, M_z=\iint_A\sigma_x y\mathrm{d}A, P_z=\iint_A\sigma_x y^2\mathrm{d}A, Q_y=\zeta\iint_A\tau_{xy}\mathrm{d}A, R_y=\zeta\iint_A\tau_{xy}y\mathrm{d}A$. 这里 ζ 为剪切修正系数.

由式(7)和式(8),不难得到用挠度、转角和损伤增量表示的梁的运动微分方程

$$\begin{cases}I_z(C_1+C_2)\otimes(\dfrac{\partial\varphi}{\partial x}\dfrac{\partial^2\varphi}{\partial x^2})\dfrac{\partial v}{\partial x}+\dfrac{1}{2}I_z(C_1+C_2)\otimes(\dfrac{\partial\varphi}{\partial x})^2\dfrac{\partial^2 v}{\partial x^2}+A(C_1+C_2)\otimes(\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x^2})\dfrac{\partial v}{\partial x}+\\ \quad\dfrac{1}{2}A(C_1+C_2)\otimes(\dfrac{\partial v}{\partial x})^2\dfrac{\partial^2 v}{\partial x^2}+\dfrac{1}{2}\zeta AC_1\otimes(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial\varphi}{\partial x})+q(x,t)=\rho A\dfrac{\partial^2 v}{\partial t^2}\\ I_z(C_1+C_2)\otimes\dfrac{\partial^2\varphi}{\partial x^2}+\dfrac{Ah^4}{60}\beta\dfrac{\partial\hat{Đ}}{\partial x}-J_z(C_1+C_2)\otimes(\dfrac{\partial\varphi}{\partial x}\dfrac{\partial^2\varphi}{\partial x^2})\dfrac{\partial\varphi}{\partial x}-\dfrac{1}{2}J_z(C_1+C_2)\otimes(\dfrac{\partial\varphi}{\partial x})^2\dfrac{\partial^2\varphi}{\partial x^2}-\\ \quad I_z(C_1+C_2)\otimes(\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x^2})\dfrac{\partial\varphi}{\partial x}-\dfrac{1}{2}I_z(C_1+C_2)\otimes(\dfrac{\partial v}{\partial x})^2\dfrac{\partial^2\varphi}{\partial x^2}-\\ \quad\dfrac{1}{2}\zeta I_zC_1\otimes[(\dfrac{\partial\varphi}{\partial x})^2+\varphi\dfrac{\partial^2\varphi}{\partial x^2}]\varphi-\dfrac{1}{2}\zeta AC_1\otimes(\dfrac{\partial v}{\partial x}+\varphi)=\rho I_z\dfrac{\partial^2\varphi}{\partial t^2}\end{cases}\tag{9}$$

其中 $I_z=\iint_A y^2\mathrm{d}A=\frac{1}{12}Ah^2, J_z=\iint_A y^4\mathrm{d}A=$

$\frac{1}{80}Ah^4$. 损伤 $\hat{D}$ 的运动微分方程为

$$-\rho k\ddot{\hat{D}} = -\alpha\frac{\partial^2\hat{D}}{\partial x^2} + \omega\dot{\hat{D}} + \xi\hat{D} + \frac{84\beta}{17h^2}\frac{\partial\varphi}{\partial x} \tag{10}$$

设梁两端是简支的,并假定端部损伤增量为零,则有边界条件为

$$v = 0,\frac{\partial\varphi}{\partial x} = 0 \quad 当\ x = 0\ 和\ x = l\ 时$$

$$\hat{D} = 0 \quad 当\ x = 0\ 和\ x = l\ 时 \tag{11}$$

给定梁的初始条件.

设粘弹性物体初始时($t < 0$)处于自然状态,且当 $t \geqslant 0$ 时满足如下初始条件

$$v\mid_{t=0} = v^0,\dot{v}\mid_{t=0} = \dot{v}^0,$$

$$\varphi\mid_{t=0} = \varphi^0,\dot{\varphi}\mid_{t=0} = \dot{\varphi}^0,\hat{D}\mid_{t=0} = 0,\dot{\hat{D}}\mid_{t=0} = \dot{\hat{D}}^0 \tag{12}$$

其中,$v^0,\dot{v}^0,\varphi^0,\dot{\varphi}^0,\dot{\hat{D}}^0$ 是已知的. 方程(9),(10)及其边界条件(11)和初始条件(12)构成控制非线性损伤粘弹性 Timoshenko 梁动力学行为的初边值问题.

3　损伤粘弹性 Timoshenko 梁的模型简化

方程组(9),(10)是一非线性的积分 - 偏微分方程组,要得出这组方程组的解析解十分困难的. 通常应用数值求解的方法,来揭示出非线性损伤粘弹性 Timoshenko 梁丰富的动力学行为.

由边界条件(11),方程(9),(10)的解可取为如下的形式

$$v(x,t) = \sum_{n=1}^{\infty}\bar{v}(t)_n\sin\frac{n\pi x}{L},$$

$$\varphi(x,t) = \sum_{n=1}^{\infty}\bar{\varphi}(t)_n\cos\frac{n\pi x}{L},$$

$$\hat{D}(x,t) = \sum_{n=1}^{\infty}\bar{\hat{D}}(t)_n\sin\frac{n\pi x}{L} \tag{13}$$

假定梁受到的横向载荷为

$$q(x,t) = \bar{q}(t)\sin\frac{\pi x}{L} \tag{14}$$

取 $n = 1,3$,将(13),(14)代入到(9),(10)中,可得简化的二阶 Galerkin 截断系统为

$$\begin{aligned}&-A_1\otimes\bar{v}_1 - A_2\otimes\bar{v}_1^2\bar{v}_1 - A_3\otimes(\bar{v}_1\bar{v}_3)\bar{v}_1 - \\&A_4\otimes\bar{v}_3^2\bar{v}_1 - A_5\otimes\bar{v}_1^2\bar{v}_3 - \\&A_6\otimes(\bar{v}_1\bar{v}_3)\bar{v}_3 - A_7\otimes\bar{\varphi}_1 - \\&A_8\otimes\bar{\varphi}_1^2\bar{v}_1 + A_9\otimes\bar{\varphi}_1^2\bar{v}_3 - \\&A_{10}\otimes(\bar{\varphi}_1\bar{\varphi}_3)v_1 - A_{11}\otimes\bar{\varphi}_3^2\bar{v}_1 + \\&\bar{q}\frac{L}{2} = A_{12}\ddot{\bar{v}}_1\end{aligned}$$

$$\begin{aligned}&-B_1\otimes\bar{v}_3 - B_2\otimes\bar{v}_1^2\bar{v}_1 - B_3\otimes(\bar{v}_1^2\bar{v}_3)\bar{v}_1 - \\&B_4\otimes\bar{v}_1^2\bar{v}_3 - B_5\otimes\bar{v}_3^2\bar{v}_3 - \\&B_6\otimes\bar{\varphi}_3 + B_7\otimes\bar{\varphi}_1^2\bar{v}_1 - \\&B_8\otimes\bar{\varphi}_1^2\bar{v}_1 - B_9\otimes\bar{\varphi}_3^2\bar{v}_3 = B_{10}\ddot{\bar{v}}_3\end{aligned}$$

$$\begin{aligned}&A_{13}\bar{\hat{D}}_1 - A_{14}\otimes\bar{v}_1 - A_{15}\otimes\bar{\varphi}_1 + A_{16}\otimes\bar{v}_1^2\bar{\varphi}_1 + \\&A_{17}\otimes(\bar{v}_1\bar{v}_3)\bar{\varphi} + A_{18}\otimes\bar{v}_3^2\bar{\varphi}_1 + \\&A_{19}\otimes\bar{v}_1^2\bar{\varphi}_3 + A_{20}\otimes\bar{\varphi}_1^2\bar{\varphi}_1 + \\&A_{21}\otimes(\bar{\varphi}_1\bar{\varphi}_3)\bar{\varphi}_1 + A_{22}\otimes\bar{\varphi}_3^2\bar{\varphi}_1 + \\&A_{23}\otimes\bar{\varphi}_1^2\bar{\varphi}_3 + A_{24}\otimes(\bar{\varphi}_1\bar{\varphi}_3)\bar{\varphi}_3 = A_{25}\ddot{\bar{\varphi}}_1\end{aligned}$$

$$\begin{aligned}&B_{11}\bar{\hat{D}}_3 - B_{12}\otimes\bar{v}_3 - B_{13}\otimes\bar{\varphi}_3 + B_{14}\otimes\bar{v}_1^2\bar{\varphi}_1 + \\&B_{15}\otimes\bar{v}_1^2\bar{\varphi}_3 + B_{16}\otimes\bar{v}_3^2\bar{\varphi}_3 + \\&B_{17}\otimes\bar{\varphi}_1^2\bar{\varphi}_1 + B_{18}\otimes(\bar{\varphi}_1\bar{\varphi}_3)\bar{\varphi}_1 + \\&B_{19}\otimes\bar{\varphi}_1^2\bar{\varphi}_3 + B_{20}\otimes\bar{\varphi}_3^2\bar{\varphi}_3 = B_{21}\ddot{\bar{\varphi}}_3\end{aligned}$$

$$A_{26}\bar{\hat{D}}_1 + A_{27}\dot{\bar{\hat{D}}}_1 - A_{28}\bar{\varphi}_1 = -A_{29}\ddot{\bar{\hat{D}}}_1$$

$$B_{22}\bar{\hat{D}}_3 + B_{23}\dot{\bar{\hat{D}}}_3 - B_{24}\bar{\varphi}_3 = -B_{25}\ddot{\bar{\hat{D}}}_3 \tag{15}$$

方程(15)中的系数与梁的几何性质和材料参数有关,由于篇幅所限,这里略去了它们的表达式. 令 $\bar{v}_3 = \bar{\varphi}_3 = \bar{\hat{D}}_{13} = \bar{\hat{D}}_{23} = 0$,即取 $n = 1$,可得一阶 Galerkin 截断简化系统.

4　问题求解

引入如下的无量纲化参量,并作如下的变量变换

$$\beta_1 = L/h, v = \bar{v}/h,$$

$$\hat{D}_1 = h^3\bar{\hat{D}}_1, \hat{D}_3 = h^3\bar{\hat{D}}_3,$$

$$\beta_2 = C_1(0)/(\rho V_c^2), \beta_3 = \beta/(\rho V_c^2),$$

$$\beta_4 = \alpha/(\rho k V_c^2), \beta_5 = \xi h^2/(\rho k V_c^2),$$

$$\beta_6 = \omega h/(\rho k V_c), \beta_7 = \beta h^2/(\rho k V_c^2),$$

$$\tau = tV_c/h, \tau_0 = t_0V_0/h,$$

$$c_1(\tau) = C_1(\tau)/C_1(0), q_0 = \bar{q}h/(AC_1(0)),$$

$$y_0 = t, y_1 = v_1, y_2 = \dot{v}_1,$$

$$y_3 = \varphi_1, y_4 = \dot{\varphi}_1, y_5 = v_3,$$

$$y_6 = \dot{v}_3, y_7 = \varphi_3, y_8 = \dot{\varphi}_3$$

$$y_9 = \int_0^t\dot{c}_1(t-\tau)v_1(\tau)\mathrm{d}\tau,$$

$$y_{10} = \int_0^t\dot{c}_1(t-\tau)v_3(\tau)\mathrm{d}\tau,$$

$$y_{11} = \int_0^t\dot{c}_1(t-\tau)\varphi_1(\tau)\mathrm{d}\tau,$$

$$y_{12} = \int_0^t \dot{c}_1(t-\tau)\varphi_3(\tau)\mathrm{d}\tau,$$

$$y_{13} = \int_0^t \dot{c}_1(t-\tau)\varphi_1^2(\tau)\mathrm{d}\tau,$$

$$y_{14} = \int_0^t \dot{c}_1(t-\tau)\varphi_1(\tau)\varphi_3(\tau)\mathrm{d}\tau,$$

$$y_{15} = \int_0^t \dot{c}_1(t-\tau)\varphi_3^2(\tau)\mathrm{d}\tau,$$

$$y_{16} = \hat{D}_1, y_{17} = \dot{\hat{D}}_1, y_{18} = \hat{D}_3$$

$$y_{19} = \dot{\hat{D}}_3, y_{20} = \int_0^t \dot{c}_1(t-\tau)v_1^2(\tau)\mathrm{d}\tau,$$

$$y_{21} = \int_0^t \dot{c}_1(t-\tau)v_1(\tau)v_3(\tau)\mathrm{d}\tau,$$

$$y_{22} = \int_0^t \dot{c}_1(t-\tau)v_3^2(\tau)\mathrm{d}\tau \tag{16}$$

对于标准线性固体,材料的松弛函数 $c_1(t)$ 满足下面的条件

$$c_1(t) = c_0 = c_1\exp(-\alpha t)$$

$$c_1(0) = c_0 + c_1 = 1$$

$$\dot{c}_1(t-\tau) = \Psi_1(t)\cdot\Psi_2(\tau) = -c_1\exp(-\alpha t)\cdot\alpha\exp(\alpha\tau) \tag{17}$$

从条件(17)和方程(15)中,我们可得下面的常微分方程组

$$\dot{Y} = F(Y) \tag{18}$$

其中 $Y = \{y_0, y_1, \cdots, y_{22}\}, F = \{F_0, F_1, \cdots, F_{22}\}$ 并有 $F_0 = 1, F_1 = y_2$,

$$F_2 = -k_1(y_1+y_9) - k_2(y_1^2+y_{20})y_1 - k_3(y_1y_5+y_{21})y_1 - k_4(y_5^2+y_{22})y_1 - k_5(y_1^2+y_{20})y_5 - k_6(y_1y_5+y_{21})y_5 - k_7(y_3+y_{11}) - k_8(y_3^2+y_{13})y_1 + k_9(y_3^2+y_{13})y_5 - k_{10}(y_3y_7+y_{14})y_1 \; k_{11}(y_7^2+y_{15})y_1 + \beta_2 q_0,$$

$$F_3 = y_4,$$

$$F_4 = k_{12}y_{16} - k_{13}(y_1+y_9) - k_{14}(y_3+y_{11}) + k_{15}(y_1^2+y_{20})y_3 - k_{16}(y_1y_5+y_{21})y_3 + k_{17}(y_5^2+y_{22})y_3 + k_{18}(y_1^2+y_{20})y_7 + k_{19}(y_3^2+y_{13})y_3 + k_{20}(y_3y_7+y_{14})y_3 + k_{21}(y_7^2+y_{15})y_3 + k_{22}(y_3^2+y_{13})y_7 + k_{23}(y_3y_7+y_{14})y_7,$$

$$F_5 = y_6,$$

$$F_6 = -k_{24}(y_5+y_{10}) - k_{25}(y_1^2+y_{20})y_1 - k_{26}(y_1y_5+y_{21})y_1 - k_{27}(y_1^2+y_{20})y_5 - k_{28}(y_5^2+y_{22})y_5 - k_{29}(y_7+y_{12}) + k_{30}(y_3^2+y_{13})y_1 - k_{31}(y_3^2+y_{13})y_5 - k_{32}(y_7^2+y_{15})y_5,$$

$$F_7 = y_8,$$

$$F_8 = k_{33}y_{18} - k_{34}(y_5+y_{10}) - k_{35}(y_7+y_{12}) + k_{36}(y_1^2+y_{20})y_3 + k_{37}(y_1^2+y_{20})y_7 + k_{38}(y_5^2+y_{22})y_7 + k_{39}(y_3^2+y_{13})y_3 + k_{40}(y_3y_7+y_{14})y_3 + k_{41}(y_3^2+y_{13})y_7 + k_{42}(y_7^2+y_{15})y_7,$$

$$F_9 = -\alpha(c_1y_1+y_9), F_{10} = -\alpha(c_1y_5+y_{10}),$$

$$F_{11} = -\alpha(c_1y_3+y_{11}), F_{12} = -\alpha(c_1y_7+y_{12}),$$

$$F_{13} = -\alpha(c_1y_3^2+y_{13}), F_{14} = -\alpha(c_1y_3y_7+y_{14}),$$

$$F_{15} = -\alpha(c_1y_7^2+y_{15}), F_{16} = y_{17},$$

$$F_{17} = -k_{43}y_{16} - k_{44}y_{17} + k_{45}y_3,$$

$$F_{18} = y_{19}, F_{19} = -k_{46}y_{18} - k_{47}y_{19} + k_{48}y_7,$$

$$F_{20} = -\alpha(c_1y_1^2+y_{20}), F_{21} = -\alpha(c_1y_1y_5+y_{21}),$$

$$F_{22} = -\alpha(c_1y_5^2+y_{22}) \tag{19}$$

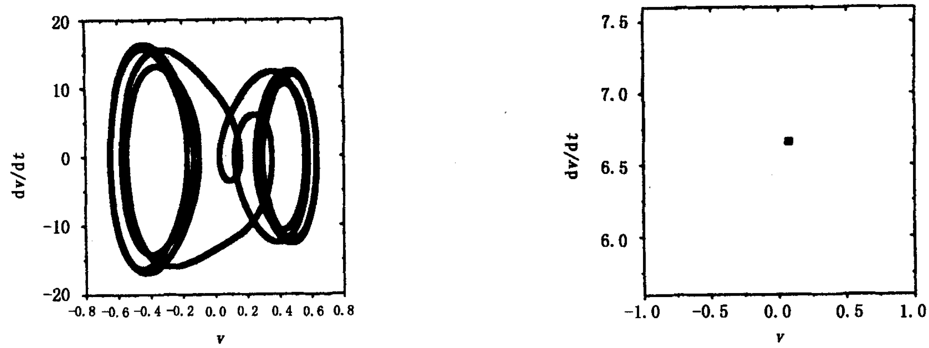
方程(19)中的系数由梁的几何性质和材料性质有关,限于篇幅,这里也略去了它们的表达式.由初始条件(12),得到方程(19)的初始条件如下

$$\{y_1(0), y_2(0), \cdots, y_{22}(0)\} = \{v_1^0, \dot{v}_1^0, \varphi_1^0, \dot{\varphi}_1^0, v_3^0, \dot{v}_3^0, \varphi_3^0, \dot{\varphi}_3^0, 0,0,0,0,0,0,0, \hat{D}_1^0, \dot{\hat{D}}_1^0, \hat{D}_3^0, \dot{\hat{D}}_3^0, 0,0,0\} \tag{20}$$

应用非线性动力学的数值分析方法,我们分析了给定截断系统(19)的动力学行为.取 $\beta_1 = 5, \beta_2 = 10^5, \beta_3 = 6.67\times10^4, \beta_4 = 3.33\times10^5, \beta_5 = 5.0\times10^3, \beta_6 = 36.1, \beta_7 = 4.17\times10^3, \zeta = 5/6, \mu = 0.23, c_1 = 0.9, q_0 = q\sin(2\pi t), \alpha = 0.2$,图(1)～图(4)分别给出了不同载荷幅值 q 的相平面图和 Poincare 截面. 同时,为了分析损伤对梁的动力学行为的影响,在相同的材料参数和载荷参数条件下,我们还分析比较了有损伤和无损伤梁的动力学性质.

从图(1)～图(4)分别给出不同的载荷参数 q 的相平面图和 Poincare 截面,从图中很容易看出,当载荷参数 q 增大时,系统由稳定的周期运动向不稳定的混沌运动转化.取 $\beta_3 = 1.334\times10^5, \beta_7 = 8.34\times10^3$,其他材料参数与上面相同.图(5)给出了有损伤梁和无损伤梁的动力学特性.可以看出,当 $q = 0.02$ 时,有损伤的梁出现混沌运动而无损

伤梁仍处于稳定的周期运动,因此,损伤可能使梁的动力学特性不稳定,对梁的动力学稳定性是有害的,因此,我们要降低有损伤有关的材料参数 $\beta_3 \sim \beta_7$,以提高梁的稳定性.

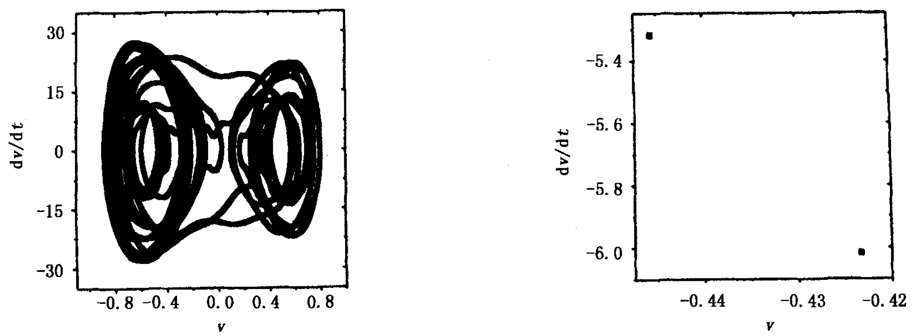


(a) 相图　　(b)Poincare 截面

(a)The phase diagram　　(b)The poincare map

图 1　$q = 0.01$ 时,系统(19) 的动力学特性

Fig. 1　The dynamical property of the system (19) as $q = 0.01$

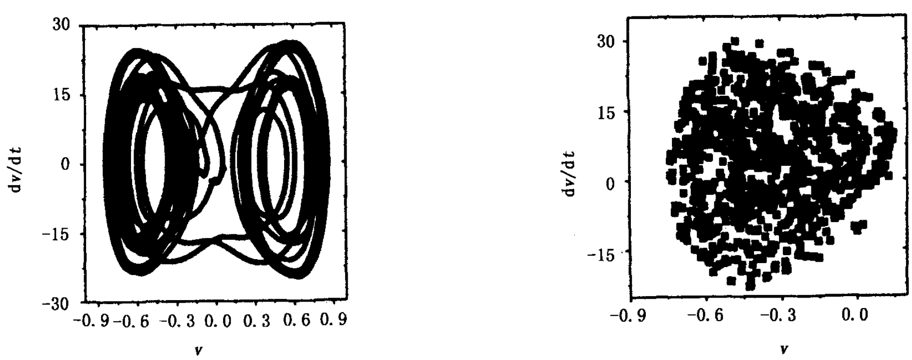


(a) 相图　　(b)Poincare 截面

(a)The phase diagram　　(b)The poincare map

图 2　$q = 0.014\,7$ 时,系统(19) 的动力学特性

Fig. 2　The dynamical property of the system (19) as $q = 0.014\,7$

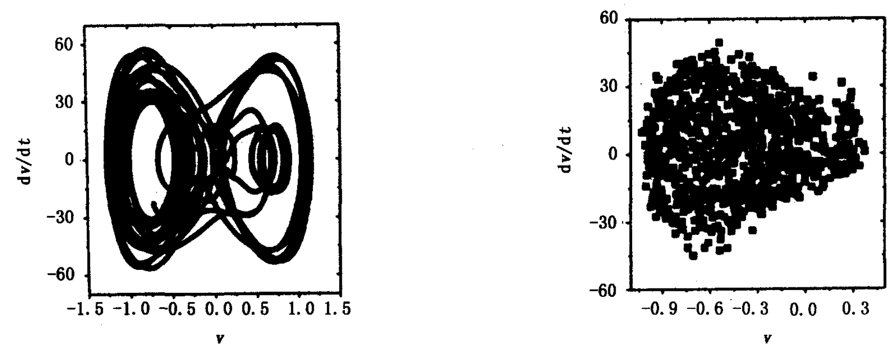


(a) 相图　　(b)Poincare 截面

(a)The phase diagram　　(b)The poincare map

图 3　$q = 0.015$ 时,系统(19) 的动力学特性

Fig. 3　The dynamical property of the system (19) as $q = 0.015$



(a) 相图

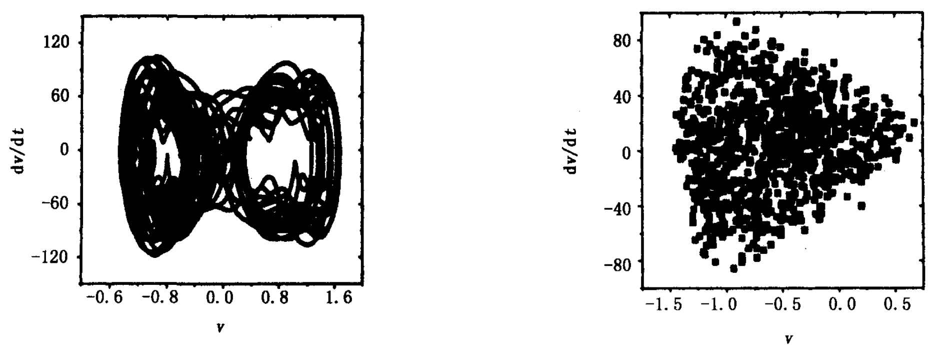
(a)The phase diagram

(b)Poincare 截面

(b)The poincare map

图4 $q = 0.025$ 时,系统(19)的动力学特性

Fig.4 The dynamical property of the system (19) as $q = 0.025$

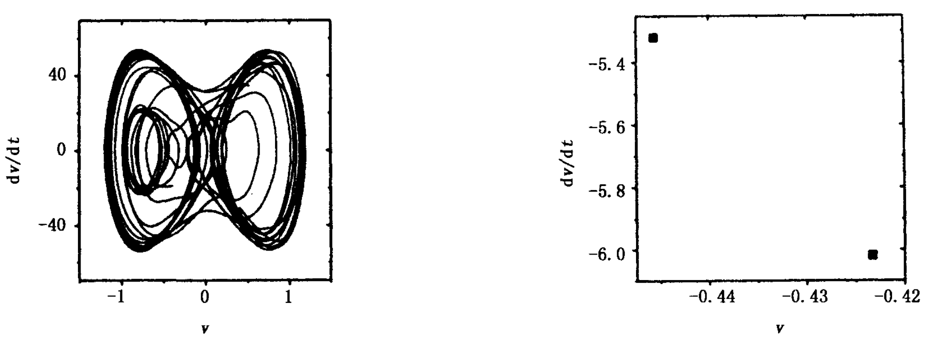


(a) 有损伤时相图

(a)The phase diagram with damage

(b) 有损伤时 Poincare 截面

(b)The poincare map with damage



(c) 无损伤时相图

(c)The phase diagram without damage

(d) 无损伤时 Poincare 截面

(d)The poincare map without damage

图5 $q = 0.05$ 时,有损伤系统和无损伤系统动力学特性的比较

Fig.5 The comparsion of dynamic properties of the system (19) with damage and without damage as $q = 0.05$

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DYNAMICAL BEHAVIORS OF NONLINEAR VISCOELASTIC TIMOSHENKO BEAMS WITH DAMAGE*

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Abstract From a convolution type constitutive model of viscoelastic solids with damage, this paper established the equations governing the static-dynamic behaviors of viscoelastic Timoshenko beams with damage under finite deflections. The Galerkin method was applied to simplify the equations, and then a set of ordinary-differential equations was obtained. The numerical methods, such as Phase-trajectory figures and Poincare sections, were used to solve the simplified system. This paper also investigated the influences of the load parameters on the dynamic behavior of nonlinear viscoelastic Timoshenko beams with damage. In particular, the effects of the damage on the dynamical behaviors of viscoelastic Timoshenko beams were considered.

Key words viscoelastic body with damage, Timoshenko beam, geometrical nonlinearity, chaos

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