六维非线性动力系统三阶规范形的计算

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摘要 首次用一种改进的共轭算子法研究了六维非线性系统的三阶规范形以及所用的非线性变换.首先简要介绍了这种改进的共轭算子法，然后通过一般形式的六维非线性系统推导出计算三阶规范形的公式.最后用MAPLE符号算法语言编写计算程序，给出了六维非线性系统的三阶规范形的具体形式.该方法可以用同一个主程序计算具有不同线性情况的六维非线性系统的三阶规范形.

关键词 规范形,共轭算子法,非线性变换,高维非线性系统,MAPLE符号语言

前言

规范形理论(Normal Form)是研究非线性动力学的主要方法之一,其主要思想是通过一系列的非线性变换将非线性系统变得尽可能简单,其实质是消去起次要作用的非线性项,而且简化后的系统与原系统拓扑等价.规范形理论不但是微分方程定性研究的重要手段,而且当它用于含参数的微分方程时也是动态分叉研究的基本工具.

由于计算量大和计算过程比较复杂，规范形理论在提出以后很长一段时间内并未得到较快的发展.直到20世纪70年代，Arnold[1]对规范形理论的发展做出了新的贡献，在他的著作中，对于规范形理论进行了详细的研究.近20年来，国内外学者对于规范形理论进行了许多卓有成效的研究.迄今为止，已提出5种方法可用于规范形的计算:矩阵表示法[2,3]、共轭算子法[4]、李代数表示论法[5]、多重李括号法[6]、多尺度摄动法[7].其中李代数表示论法由于涉及到较为高深的数学知识，在实际的工程中并未得到较大应用.

近年来，国内外许多学者开始关注对高维非线性系统规范形计算的研究.Zhang[8]利用矩阵表示法计算了具有Z_2-对称性的非线性动力系统的五阶规范形.Leung和Zhang等人[9]用Mathematica软件将规范形方法与其他方法共同使用,得到了多维非线性系统规范形的系数,并与其他方法所得到的结果进行了比较.Li等人[10]改进了由Kokubu等人提出的方法,并对Baider和Sanders[11]论文中关于最简规范形的公开问题作了解答,解决了一般情况下Bogdanov-Takens向量场的最简规范形问题.Yu[12]利用多尺度方法计算了非线性动力系统的规范形.最近，Zhang和Wang等人提出了一种改进的共轭算子法，并计算了3种线性情况下四维非线性系统的三阶规范形，利用MAPLE语言编写了计算程序，而且还将这种方法应用于非线性非平面运动悬臂梁平均方程的规范形的计算.

1 规范形理论和共轭算子法

考虑微分方程

$$\dot{x}=X(x)=\boldsymbol{A}(x)+f^2(x)+\cdots+f^r(x)+O(|x|^r)\quad x\in\boldsymbol{R}^n,\tag{1}$$

这里$\boldsymbol{A}$是$n\times n$ Jordan标准形矩阵,$f^k(x)\in H_n^k(k=2,\cdots,r)$,$H_n^k$表示所有$n$维$k$次齐次多项式组成的线性空间.假定原点$x=0$是一个奇点，则有$X(0)=0$.下面用一系列近恒同的非线性变换，将系统(1)的非线性项化简成简单的形式。

令近恒同的非线性变换为

$$x=y+\boldsymbol{P}^k(y),\quad \boldsymbol{P}^k(y)\in H_n^k,k=2,\cdots,r\tag{2}$$

则有

$$\dot{x}=(I+\mathrm{D}\boldsymbol{P}^k(y))\dot{y}$$
$$(I+\mathrm{D}\boldsymbol{P}^k(y))^{-1}=I-\mathrm{D}\boldsymbol{P}^k(y)+\cdots\tag{3}$$

将方程(2)和(3)代入系统(1)，得

$$\dot{y}=\boldsymbol{A}y+f^2(y)+\cdots+f^{k-1}(y)+f^k(y)-[\mathrm{D}\boldsymbol{P}^k(y)\boldsymbol{A}y-\boldsymbol{A}\boldsymbol{P}^k(y)]+O(|y|^{k+1})\tag{4}$$

定义一个线性算子$ad_A^k:H_n^k\rightarrow H_n^k$，即

$$ad_A^k\boldsymbol{P}^k(y)=\mathrm{D}\boldsymbol{P}^k(y)\boldsymbol{A}y-\boldsymbol{A}\boldsymbol{P}^k(y)\quad \boldsymbol{P}^k(y)\in H_n^k\tag{5}$$

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令 R_n^k 是ad_A^k 的值域，$R_n^k = \mathrm{Im} ad_A^k$，$C_n^k$ 是R_n^k 在 H_n^k 中的任意补空间，$H_n^k = R_n^k \oplus C_n^k$. 于是有 $h^k \in R_n^k, g^k(y) \in C_n^k, f^k(y)$ 可以表示成

$$f^k(y) = h^k(y) + g^k(y) \tag{6}$$

如果我们适当选择 $\boldsymbol{P}^k(y)$ 使得 $ad_A^k \boldsymbol{P}^k(y) = h^k(y)$，则(4) 式成为

$$\dot{y} = \boldsymbol{A}y + f^2(y) + \cdots + f^{k-1}(y) + g^k(y) + O(|y|^{k+1}) \tag{7}$$

上述讨论对于任意自然数 $2 \leqslant k \leqslant r$ 都成立. 因此可以通过一系列的坐标变换，依次使得各阶非线性项 f^i 化简为 $g^i, g^i \in G_n^i (i = 2, \cdots, r)$.

下面给出一个关于共轭算子法的定理.

定理 设 V 为一有限维内积空间，Γ 为 V 上的线性算子，Γ^* 为 Γ 的共轭算子，则有

1) $\mathrm{Ker}\Gamma^* = (\mathrm{Im}\Gamma)^{\perp}$，2) $V = \mathrm{Im}\Gamma \oplus \mathrm{Ker}\Gamma^*$

这里 $\mathrm{Ker}\Gamma^*$ 为 Γ^* 的零空间。

从这个定理可以看出共轭算子的零空间是线性算子值域的垂直补空间. 从方程(5) 可以看出线性算子 $ad_A^k : H_n^k \to H_n^k$ 的共轭算子为ad_A^{k*}. 由于

$$ad_A^k \boldsymbol{P}^k(y) = \mathrm{D}\boldsymbol{P}^k(y)\boldsymbol{A}y - \boldsymbol{A}\boldsymbol{P}^k(y)$$
$$\boldsymbol{P}^k(y) \in H_n^k$$

所以有

$$ad_A^{k*} \boldsymbol{P}^k(y) = \mathrm{D}\boldsymbol{P}^k(y)\boldsymbol{A}^* y - \boldsymbol{A}^* \boldsymbol{P}^k(y)$$
$$\boldsymbol{P}^k(y) \in H_n^k \tag{8}$$

共轭算子的零空间即为系统(1) 值域的垂直补空间，即有

$$H_n^k = \mathrm{Ker}(ad_A^{k*}) \oplus R_n^k \tag{9}$$

下列方程所确定的空间即为我们所求的 C_n^k 空间，即有

$$\mathrm{D}\boldsymbol{P}^k(y)\boldsymbol{A}^* y - \boldsymbol{A}^* \boldsymbol{P}^k(y) = 0 \tag{10}$$

该方程被称之为共轭算子方程，方程(10) 是一个线性偏微分方程，求规范形关键在于如何求解该方程.

在 Elphick 等人[4] 的论文中，为了找到该偏微分方程组的所有 k 次多项式解，研究者们是通过偏微分方程的特征方程及其首次积分来求解的. 当非线性系统的维数比较高时，求解特征方程和首次积分是一项困难的工作. 在文献[13] 中，Zhang 等人提出了一种改进的共轭算子法. 在这种改进的共轭算子法中，通过在 H_n^k 中引入一组单项式基后，利用求解高维非线性和线性代数方程组的方法来得到所有三次多项式解. 下面利用改进的共轭算子法研究一个六维非线性系统，推导出计算规范形的公式.

2 改进的共轭算子法和应用

在本节中，我们只考虑系统六维非线性系统三阶规范形的计算。一般的六维非线性系统可以表示成下面的形式

$$\dot{x} = X(x) = \boldsymbol{A}(x) + f^3(x)$$
$$x \in \boldsymbol{R}^6, \tag{11}$$

这里 $f^3(x) \in H_6^3$.

$$f^3(x) = (f_1^3(x), f_2^3(x), \cdots, f_5^3(x), f_6^3(x))^{\mathrm{T}} = \Big(\sum_{|m|=3} a_{m_1 m_2 \cdots m_6}\big(\prod_{i=1}^{6} x_i^{m_i}\big), \cdots, \sum_{|m|=3} f_{m_1 m_2 \cdots m_6}\big(\prod_{i=1}^{6} x_i^{m_i}\big)\Big)^{\mathrm{T}}$$

这里 $|m| = m_1 + m_2 + m_3 + m_4 + m_5 + m_6$.

在这种情况下，线性算子为

$$ad_A^k \boldsymbol{P}^k(y) = \mathrm{D}\boldsymbol{P}^3(x)\boldsymbol{A}x - \boldsymbol{A}\boldsymbol{P}^3(x) \tag{12}$$

共轭算子方程(10) 变为

$$\mathrm{D}\boldsymbol{P}^3(x)\boldsymbol{A}^* x - \boldsymbol{A}^* \boldsymbol{P}^3(x) = 0 \tag{13}$$

一般来说，在六维非线性系统中，Jordan 标准形矩阵 $\boldsymbol{A}$ 有以下 4 种情况：

1) $\boldsymbol{A}$ 有三对纯虚特征值；

2) $\boldsymbol{A}$ 有一对双零特征值，两对纯虚特征值；

3) $\boldsymbol{A}$ 有两对双零特征值，一对纯虚特征值；

4) $\boldsymbol{A}$ 有三对双零特征值.

在本文里，我们只研究情况(1)，即

$$\boldsymbol{A} = \begin{bmatrix} 0 & -\omega_1 & 0 & 0 & 0 & 0 \\ \omega_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega_2 & 0 & 0 \\ 0 & 0 & \omega_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega_3 \\ 0 & 0 & 0 & 0 & \omega_3 & 0 \end{bmatrix} \tag{14}$$

情况(1) 又可分为共振和非共振两种情况，我们只研究非共振的情况，即

$$\omega_1 \neq \omega_2 \neq \omega_3$$

$\boldsymbol{P}(x)$ 的 Jacobi 矩阵为

$$\mathrm{D}\boldsymbol{P}^3(x) = \Big\{\frac{\partial \boldsymbol{P}^3(x)}{\partial x}\Big\}_{6\times 6} = \begin{bmatrix} \frac{\partial P_1^3}{\partial x_1} & \frac{\partial P_1^3}{\partial x_2} & \frac{\partial P_1^3}{\partial x_3} & \frac{\partial P_1^3}{\partial x_4} & \frac{\partial P_1^3}{\partial x_5} & \frac{\partial P_1^3}{\partial x_6} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial P_6^3}{\partial x_1} & \frac{\partial P_6^3}{\partial x_2} & \frac{\partial P_6^3}{\partial x_3} & \frac{\partial P_6^3}{\partial x_4} & \frac{\partial P_6^3}{\partial x_5} & \frac{\partial P_6^3}{\partial x_6} \end{bmatrix}_{6\times 6} \tag{15}$$

将共轭转置矩阵 $\boldsymbol{A}^*$ 和(15)代入方程(13),得到以下线性偏微分方程组

$$\omega_1 x_2 \frac{\partial P_1^3}{\partial x_1} - \omega_1 x_1 \frac{\partial P_1^3}{\partial x_2} + \omega_2 x_4 \frac{\partial P_1^3}{\partial x_3} - \omega_2 x_3 \frac{\partial P_1^3}{\partial x_4} + \omega_3 x_6 \frac{\partial P_1^3}{\partial x_5} - \omega_3 x_5 \frac{\partial P_1^3}{\partial x_6} - \omega_1 P_2^3 = 0 \quad (16a)$$

$$\omega_1 x_2 \frac{\partial P_2^3}{\partial x_1} - \omega_1 x_1 \frac{\partial P_2^3}{\partial x_2} + \omega_2 x_4 \frac{\partial P_2^3}{\partial x_3} - \omega_2 x_3 \frac{\partial P_2^3}{\partial x_4} + \omega_3 x_6 \frac{\partial P_2^3}{\partial x_5} - \omega_3 x_5 \frac{\partial P_2^3}{\partial x_6} + \omega_1 P_1^3 = 0 \quad (16b)$$

…………,

$$\omega_1 x_2 \frac{\partial P_8^3}{\partial x_1} - \omega_1 x_1 \frac{\partial P_8^3}{\partial x_2} + \omega_2 x_4 \frac{\partial P_8^3}{\partial x_3} - \omega_2 x_3 \frac{\partial P_8^3}{\partial x_4} + \omega_3 x_6 \frac{\partial P_8^3}{\partial x_5} - \omega_3 x_5 \frac{\partial P_8^3}{\partial x_6} + \omega_3 P_5^3 = 0 \quad (16f)$$

为了求得偏微分方程组(16)的三次多项式解,在六维空间中引入以下56个三次齐次单项式

$$\boldsymbol{X} = [x_1^3, \cdots, x_6^3, \underbrace{x_1^2 x_2, \cdots, x_6^2 x_5}_{30}, \underbrace{x_1 x_2 x_3, \cdots, x_4 x_5 x_6}_{20}] \quad (17)$$

显然,利用上述56个单项式可以把六维线性空间中所有的三次齐次多项式表示出来.由此易得

$$f^3(x) = [C_{6\times56}]\boldsymbol{X}^{\mathrm{T}} \quad (18)$$

这里系数矩阵$[C_{6\times56}]$中的336个元素由系统(11)中的非线性项的系数确定.

此外,还有

$$\boldsymbol{P}^3(x) = [P_1^3, P_2^3, P_3^3, P_4^3, P_5^3, P_6^3] = [\sum_{|m|=3} d_{1m}x^m, \sum_{|m|=3} d_{2m}x^m, \cdots, \sum_{|m|=3} d_{6m}x^m] = [D_{6\times56}]\boldsymbol{X}^{\mathrm{T}} \quad (19)$$

这里 $m = m_1 m_2 m_3 m_4 m_5 m_6, x^m = x_1^{m_1} x_2^{m_2} x_3^{m_3} x_4^{m_4} x_5^{m_5} x_6^{m_6}$, $[D_{6\times56}]$是未知系数矩阵,即近恒同非线性变换的系数矩阵.

将(17)和(19)两式代入方程(16),得到以下方程组

$$[\omega_1 x_2 \frac{\partial}{\partial x_1} - \omega_1 x_1 \frac{\partial}{\partial x_2} + \omega_2 x_4 \frac{\partial}{\partial x_3} - \omega_2 x_3 \frac{\partial}{\partial x_4} + \omega_3 x_6 \frac{\partial}{\partial x_5} - \omega_3 x_5 \frac{\partial}{\partial x_6}] \times \sum d_{1m}x^m - \omega_1 \sum d_{2m}x^m = 0 \quad (20a)$$

…………,

$$[\omega_1 x_2 \frac{\partial}{\partial x_1} - \omega_1 x_1 \frac{\partial}{\partial x_2} + \omega_2 x_4 \frac{\partial}{\partial x_3} - \omega_2 x_3 \frac{\partial}{\partial x_4} + \omega_3 x_6 \frac{\partial}{\partial x_5} - \omega_3 x_5 \frac{\partial}{\partial x_6}] \times \sum d_{6m}x^m + \omega_3 \sum d_{5m}x^m = 0 \quad (20f)$$

将上面六个方程化简,比较同类项系数可得到$[D_{6\times56}]$中的各个系数.将$[D_{6\times56}]$代入方程(12),得到

$$[-\omega_1 x_2 \frac{\partial}{\partial x_1} + \omega_1 x_1 \frac{\partial}{\partial x_2} - \omega_2 x_4 \frac{\partial}{\partial x_3} + \omega_2 x_3 \frac{\partial}{\partial x_4} - \omega_3 x_6 \frac{\partial}{\partial x_5} + \omega_3 x_5 \frac{\partial}{\partial x_6}] \times \sum d_{1m}x^m + \omega_1 \sum d_{2m}x^m = 0 \quad (21a)$$

…………,

$$[-\omega_1 x_2 \frac{\partial}{\partial x_1} + \omega_1 x_1 \frac{\partial}{\partial x_2} - \omega_2 x_4 \frac{\partial}{\partial x_3} + \omega_2 x_3 \frac{\partial}{\partial x_4} - \omega_3 x_6 \frac{\partial}{\partial x_5} + \omega_3 x_5 \frac{\partial}{\partial x_6}] \times \sum d_{6m}x^m - \omega_3 \sum d_{5m}x^m = 0 \quad (21f)$$

简化方程(21),得到

$$h^3(x) = [K_{6\times56}]\boldsymbol{X}^{\mathrm{T}} \quad (22)$$

将(18)和(22)式代入方程(6),得到

$$[C_{6\times56}]\boldsymbol{X}^{\mathrm{T}} = [E_{6\times56}]\boldsymbol{X}^{\mathrm{T}} + [K_{6\times56}]\boldsymbol{X}^{\mathrm{T}} \quad (23)$$

平衡方程(23)两边同类项的系数,可以得到$[E_{6\times56}]$,即规范形的系数.

3　MAPLE计算程序及所得规范形

利用上述一些计算公式,我们可以利用MAPLE符号语言编写具体的计算规范形的程序.编写计算程序的主要框图如下:

1) 构造矩阵$[C_{6\times56}]$

2) 给出近恒同变换 $\boldsymbol{P}^3(x) = [C_{6\times56}]\boldsymbol{X}^{\mathrm{T}}$

3) 将 $\boldsymbol{A}^*, \boldsymbol{P}^3(x), \mathrm{D}\boldsymbol{P}^3(x)$ 带入共轭算子方程,比较系数得到$[C_{6\times56}]$

4) 将 $\boldsymbol{A}^*, \boldsymbol{P}^3(x), \mathrm{D}\boldsymbol{P}^3(x)$ 带入线性算子,得到$[C_{6\times56}]$

5) 得到规范形的系数$[C_{6\times56}]$

通过MAPLE程序运算可得系统(11)的三阶规范形如下

$$\dot{x}_1 = a_1 x_2^3 + a_2 x_1^3 + a_3 x_2 x_4^2 + a_4 x_1 x_4^2 + a_5 x_2 x_3^2 + a_6 x_1 x_3^2 + a_7 x_1 x_2^2 + a_8 x_1^2 x_2 + a_9 x_2 x_6^2 + a_{10} x_1 x_6^2 +$$

$$a_{11}x_2x_5^2 + a_{12}x_1x_5^2$$

$$\dot{x}_2 = b_1x_2^3 + b_2x_1^3 + b_3x_1^2x_2 + b_4x_2x_6^2 + b_5x_1x_6^2 + b_6x_2x_5^2 + b_7x_1x_5^2 + b_8x_2^2x_4 + b_9x_1x_4^2 + b_{10}x_2x_3^2 + b_{11}x_1x_3^2 + b_{12}x_1x_2^2$$

$$\dot{x}_3 = c_1x_3^3 + c_2x_4^3 + c_3x_3^2x_4 + c_4x_4x_6^2 + c_5x_3x_6^2 + c_6x_4x_5^2 + c_7x_3x_5^2 + c_8x_2^3x_4 + c_9x_2x_4^2 + c_{10}x_2^2x_3 + c_{11}x_1^2x_4 + c_{12}x_1^2x_3$$

$$\dot{x}_4 = d_1x_4^3 + d_2x_3^2 + d_3x_3x_6^2 + d_4x_1^2x_3 + d_5x_4x_6^2 + d_6x_4x_5^2 + d_7x_3x_5^2 + d_8x_3x_4^2 + d_9x_3^2x_4 + d_{10}x_2^2x_4 + d_{11}x_2^2x_3 + d_{12}x_1^2x_4$$

$$\dot{x}_5 = e_1x_5^3 + e_2x_6^3 + e_3x_2^2x_5 + e_4x_1^2x_6 + e_5x_1^2x_5 + e_6x_5x_6^2 + e_7x_5^2x_6 + e_8x_4^2x_6 + e_9x_4^2x_5 + e_{10}x_3^2x_6 + e_{11}x_3^2x_5 + e_{12}x_2^2x_6$$

$$\dot{x}_6 = g_1x_5^3 + g_2x_6^3 + g_3x_1^2x_6 + g_4x_1^2x_5 + g_5x_3^2x_5 + g_6x_2x_6^2 + g_7x_2^2x_5 + g_8x_5^2x_6 + g_9x_5^2x_6 + g_{10}x_4^2x_6 + g_{11}x_4^2x_5 + g_{12}x_3^2x_6$$

这里

$$a_1 = a_8 = \frac{1}{8}f_{1210000} + \frac{3}{8}f_{1030000} - \frac{3}{8}f_{2300000} - \frac{1}{8}f_{2120000}$$

$$a_2 = a_7 = \frac{1}{8}f_{2210000} + \frac{3}{8}f_{1300000} + \frac{1}{8}f_{1120000} + \frac{3}{8}f_{2030000}$$

$$a_3 = a_5 = \frac{1}{4}f_{1012000} - \frac{1}{4}f_{2102000} - \frac{1}{4}f_{2100200} + \frac{1}{4}f_{1010200}$$

$$a_4 = a_6 = \frac{1}{4}f_{1100200} + \frac{1}{4}f_{2010200} + \frac{1}{4}f_{2012000} + \frac{1}{4}f_{1102000}$$

$$a_9 = a_{11} = -\frac{1}{4}f_{2100002} + \frac{1}{4}f_{1010002} - \frac{1}{4}f_{2100020} + \frac{1}{4}f_{1010020}$$

$$a_{10} = a_{12} = \frac{1}{4}f_{1100002} + \frac{1}{4}f_{2010002} + \frac{1}{4}f_{2010020} + \frac{1}{4}f_{1100020}$$

$$g_1 = g_8 = \frac{1}{8}f_{6000012} - \frac{1}{8}f_{5000003} - \frac{1}{8}f_{5000021} + \frac{1}{8}f_{6000030}$$

$$g_2 = g_9 = \frac{1}{8}f_{6000021} + \frac{3}{8}f_{5000030} + \frac{1}{8}f_{5000012} + \frac{3}{8}f_{6000003}$$

$$g_3 = g_6 = \frac{1}{4}f_{6200001} + \frac{1}{4}f_{6020001} + \frac{1}{4}f_{5200010} + \frac{1}{4}f_{5020010}$$

$$g_4 = g_7 = \frac{1}{4}f_{6200010} - \frac{1}{4}f_{5020001} - \frac{1}{4}f_{5200001} + \frac{1}{4}f_{6020010}$$

$$g_5 = g_{11} = \frac{1}{4}f_{6000210} - \frac{1}{4}f_{5002001} + \frac{1}{4}f_{6002010} - \frac{1}{4}f_{5000201}$$

$$g_{10} = g_{12} = \frac{1}{4}f_{6000201} - \frac{1}{4}f_{5000210} - \frac{1}{4}f_{6002001} + \frac{1}{4}f_{5002010}$$

这里 $f_{ni\ldots mn}$ 为系统(11) 中相应的非线性项的系数.

4 结论

在本文里，我们首次利用改进的共轭算子法计算了六维非线性动力系统的三阶规范形以及所用的非线性变换. 利用这种方法，只用一个 MAPLE 主程序就可以计算出在四种线性部分情况下,六维非线性系统的三阶规范形. 当我们研究非线性系统在不同共振情况的全局动力学问题时，上述方法相当方便.

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COMPUTATION OF THE THIRD ORDER NORMAL FORM FOR SIX-DIMENSIONAL NONLINEAR DYNAMICAL SYSTEMS

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Abstract An improved adjoint operator method was proposed to compute the third order normal form of six dimensional nonlinear dynamical systems and the associated nonlinear transformation for the first time. First the improved adjoint operator method was briefly introduced. Then, a general six dimensional nonlinear system was analyzed to derive the formula of computing the third order normal form. Finally, the MAPLE symbolic program for calculating the third order normal form was given. The concrete normal form of six dimensional nonlinear systems was obtained. The results indicate that we may respectively obtain the normal forms, the coefficients of the normal forms and the associated nonlinear transformations for six dimensional nonlinear systems in four different cases by using the same main MAPLE symbolic program.

Key words normal form, adjoint operator method, nonlinear transformation, high dimensional nonlinear systems, MAPLE program

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