第 2 卷第 3 期
2004 年 9 月

动 力 学 与 控 制 学 报
JOURNAL OF DYNAMICS AND CONTROL

Vol. 2 No. 3
Sep. 2004

扁锥面单层网壳的非线性动力学特性*

王新志　梁从兴　栗蕾　韩明君　丁雪兴
(兰州理工大学理学院，兰州　730050)

摘要　用拟壳法建立了正三角形网格三向扁锥面单层网壳的轴对称非线性动力学基本方程. 通过分离变量函数法，用 Galerkin 法得到了一个含二次、三次的非线性微分方程. 为了研究混沌运动，对一类非线性动力系统的自由振动方程进行了求解，给出了单层扁锥面网壳非线性自由振动微分方程的准确解. 通过求 Melnikov 函数，给出了发生混沌运动的临界条件. 数字仿真证实了混沌运动的存在.

关键词　单层网壳，分离变量，临界条件，混沌

引言

网壳结构由于它具有受力合理，重量轻，造价低. 目前在大中跨度覆盖建筑中得到了广泛的应用，由于大地震的频繁发生，地球表面经常遭受狂风暴雨、暴雪的袭击，这些结构时有遭到破坏. 这种建筑结构动态稳定性问题自然也就提出来了. 国内外学者对这方面都有探讨[1~8]，但对其分岔、混沌运动研究得很少. 我们在文[8]中对网架结构的分岔和混沌已开始进行探讨，本文是文[8]的继续.

1　物理方程

正三角形网格三向格子性扁锥面网壳等效刚度近似取[9]

$$T_{rr}=T_{\theta\theta}=\frac{9}{8}\frac{EA}{b},T_{r\theta}=T_{\theta r}=\frac{3}{8}\frac{EA}{b},$$

$$B_{rr}=B_{\theta\theta}=\frac{9}{8}\frac{EI}{b},B_{r\theta}=B_{\theta r}=\frac{3}{8}\frac{EI}{b},$$

$$N_r=T_{rr}\varepsilon_r+T_{r\theta}\varepsilon_\theta,N_\theta=T_{\theta\theta}\varepsilon_\theta+T_{\theta r}\varepsilon_r,$$

$$M_r=B_{rr}\chi_r+B_{r\theta}\chi_\theta,M_\theta=B_{\theta\theta}\chi_\theta+B_{\theta r}\chi_r,$$

$$\chi_r=-\frac{\partial^2 w}{\partial r^2},\chi_\theta=-\frac{1}{r}\frac{\partial w}{\partial r},$$

$$\varepsilon_r=\frac{\partial u}{\partial r}+\frac{H}{R}\frac{\partial w}{\partial r}+\frac{1}{2}(\frac{\partial w}{\partial r})^2,\varepsilon_\theta=\frac{u}{r}$$

其中 E 为弹性模量，A 为杆的横截面面积，I 为杆的惯性矩，l 为杆长，$b=\frac{\sqrt{2}}{2}l$，u 为径向位移，w 为横向位移，H 为锥壳拱高，R 为锥壳底半径.

2　基本方程和边界条件

由薄壳非线性动力学理论

$$\frac{9RI}{8a}L(w)=q+\frac{1}{r}\frac{\partial}{\partial r}\left[rN_r\left(\frac{H}{R}+\frac{\partial w}{\partial r}\right)\right]-c\frac{\partial w}{\partial t}-\gamma\frac{\partial^2 w}{\partial t^2}\tag{1}$$

$$\frac{b}{EA}r\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r^2N_r)=-\frac{H}{R}\frac{\partial w}{\partial r}-\frac{1}{2}(\frac{\partial w}{\partial r})^2\tag{2}$$

边界条件为

$$当\ r=R,w=\frac{\partial w}{\partial r}=0\tag{3}$$

$$当\ r=0,w,\frac{\partial w}{\partial r},N_r\ 有限\tag{4}$$

其中 $L=\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}$，$\gamma$ 为单位面积体质量，c 为阻尼系数.

引入无量纲量

$$\rho=\frac{r}{R},W=\frac{w}{\beta},Q=\frac{8bR^4q}{9E\beta^5},$$

$$N=\frac{8bR_rN_r}{9E\beta^5},\bar{\gamma}=\frac{8bR^4}{9EI}\gamma,\bar{c}=\frac{8bR^4}{9EI}c,$$

$$K=\frac{H}{\beta},I=\beta^4$$

则方程(1～4)简化为

$$L_1(w)=Q+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[N\left(K+\frac{\partial w}{\partial\rho}\right)\right]-\bar{c}\frac{\partial w}{\partial t}-\bar{\gamma}\frac{\partial^2 w}{\partial t^2}\tag{5}$$

$$L_2(\rho N)=-\alpha\left(K+\frac{1}{2}\frac{\partial W}{\partial\rho}\right)\frac{\partial W}{\partial\rho}\tag{6}$$

$$当\ \rho=0,w,\frac{\partial w}{\partial\rho},N\ 有限\tag{7}$$

$$当\ \rho=1,W=\frac{\partial W}{\partial\rho}=0,\frac{\partial N}{\partial\rho}-\frac{1}{3}N=0\tag{8}$$

2004-08-01 收到第 1 稿，2004-09-02 收到修改稿
* 甘肃省自然科学基金资助项目(ZS021-A25-007-Z)

这里,算子 $L_1 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}$,$L_2 = \rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}$,对圆杆 $\alpha = \frac{4A}{9\beta^2} = 1.575\,5$.

取 $W = f(t)(1-\rho^2)^2$　(9)

将式(9)代入式(6),由边界条件得 N,再将 N 和式(9)代入式(5)通过 Galerkin 作用,可得一个含二次、三次项的非线性微分方程

$$\frac{\mathrm{d}^2 f}{\mathrm{d}t^2} + \omega^2 f - \alpha_1 f^2 + \alpha_2 f^3 = g\cos\Omega t - C_1\frac{\mathrm{d}f}{\mathrm{d}t} \tag{10}$$

其中 $\omega^2 = \frac{10}{\gamma}\left(\frac{32}{3} + 0.938\,4k^2\right)$,$C_1 = \frac{\bar{c}}{\gamma}$,$\alpha_1 = \frac{13.320k}{\bar{\gamma}}$,$\alpha_2 = \frac{1.835}{\bar{\gamma}}$,$g\cos\Omega t = \frac{5\bar{Q}}{3\gamma}$,

取 $\tau = \omega t$,$f = \frac{\omega}{\sqrt{\alpha_2}}x$

由式(10)可得

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + x - \beta_2 x^2 + x^3 = F\cos\frac{\Omega}{\omega}\tau - \beta_0\frac{\mathrm{d}x}{\mathrm{d}\tau} \tag{11}$$

其中 $\beta_0 = \frac{C_1}{\omega}$,$\beta_2 = \frac{\alpha_1}{\omega\sqrt{\alpha_2}}$,$F = \frac{g\sqrt{\alpha_2}}{\omega^3}$

方程(11)不考虑外激励,其等价系统为

$$\begin{cases} x_1' = x_2 \\ x_2' = -x_1 + \beta_2 x_1^2 - x_1^3 - \beta_0 x_2 \end{cases}$$

此系统三个平衡点为(0,0),$\left(\frac{\beta_2}{2} \pm \sqrt{\frac{\beta_2^2}{4} - 1}, 0\right)$,在平衡点(0,0)处,Floguet 指数为

$$\lambda_{1,2} = -\frac{\beta_0}{2} \pm \sqrt{\frac{\beta_0^2}{2} - 1}$$

当 $\beta_0 \geqslant 2$ 时,则此平衡点为稳定的结点;当 $0 < \beta_0 < 2$ 时,λ 离开数轴在复平面上,有稳定的结点;当 $\beta_0 = 0$ 时,λ 是纯虚数,解的曲线是封闭的,即极限环.这时发生 Hopf 分岔,若有必要,其它两个不动点也可求出 Floquet 指数.

3　一类非线性动力系统自由振动方程的解

考虑方程

$$x'' + \beta_1 x - \beta_2 x^2 + \beta_3 x^3 = 0,\quad (\beta_1, \beta_2, \beta_3 > 0) \tag{12}$$

通过求此方程同宿轨道可得解为

$$x = \frac{2\beta_1}{\frac{2}{3}\beta_2 \pm \sqrt{\left(\frac{2}{3}\beta_2\right)^2 - 2\beta_1\beta_3}\sin\sqrt{\beta_1}(\tau + c)}$$

$$\left(\beta_2^2 > \frac{9}{2}\beta_1\beta_3\right) \tag{13}$$

由方程(11)知 $\beta_1 = \beta_3 = 1$,则得(12)的自由振动方程的解为 $x_{1,2} = \frac{2}{a \pm b\sin(\tau + c)}$

其中 $a = \frac{2}{3}\beta_2$,$b = \sqrt{a^2 - 2}$

取 $c = 0$,初始条件得到满足(位移不等于零,速度不等于零).则

$$x_{1,2} = \frac{2}{a \pm b\sin\tau} \tag{14}$$

取 $c = n\pi + \frac{\pi}{2}$　$(n = 0,1,2,3,\cdots)$ 得

$$x_{1,2} = \frac{2}{a \pm b\cos\tau} \tag{15}$$

4　Melnikov 函数的留数的计算

为了书写方便,方程(13)以 t 代 τ,以 ω 代替 $\frac{\Omega}{\omega}$

4.1　初始条件 1

由方程(14)可取

$$x(t) = \frac{2}{a - b\sin t},$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y(t) = \frac{2b\cos t}{(a - b\sin t)^2}$$

由定义的 Melnikov 函数[10]

$$M(t_0) = \int_{-\infty}^{+\infty}[-\beta_0 y^2(t) + Fy(t)\cos\omega(t + t_0)]\mathrm{d}t = -4\beta_0 b^2\int_{-\infty}^{+\infty}\frac{\cos^2 t}{(a - b\sin t)^4}\mathrm{d}t + 2bF\int_{-\infty}^{+\infty}\frac{\cos\cos\omega(t + t_0)}{(a - b\sin t)^2}\mathrm{d}t \tag{16}$$

这里需要计算积分

$$V_1 = \int_{-\infty}^{+\infty}\frac{\cos^2 t}{(a - b\sin t)^4}\mathrm{d}t$$

$$V_2 = \int_{-\infty}^{+\infty}\frac{\cos\cos\omega(t + t_0)}{(a - b\sin t)^2}\mathrm{d}t$$

令 $z = \mathrm{e}^{it}$,则 $\cos t = \frac{z^2 + 1}{2z}$,$\sin t = \frac{z^2 - 1}{2\mathrm{i}z}$,

$\cos\omega(t - t_0) = \frac{z^{2\omega}(\mathrm{e}^{\mathrm{i}\omega t} + \mathrm{e}^{-\mathrm{i}\omega t_0})}{2z^{\omega}}\mathrm{d}t = \frac{\mathrm{d}z}{\mathrm{i}z}$

1) 对 $V_1 = -\int_c \frac{\mathrm{i}4z(z^2 + 1)^2}{b^4\left(z^2 - \frac{2a\mathrm{i}}{b}z - 1\right)^4}\mathrm{d}z$

被积函数有极点 $\frac{a \pm \sqrt{2}}{b}i$,且是四级极点.

这里 c 是 $|z = 1|$ 的圆周,在 $|z| = 1$ 内只有一个四级极点,$z = \frac{a - \sqrt{2}}{b}\mathrm{i}$

通过留数计算可得

$$V_1=-\frac{\pi a}{4\sqrt{2}} \tag{17}$$

2) 对 $V_2=-\int_c \frac{i(z^2+1)(z^{2\omega}e^{i\omega t_0})}{b^2 z^{\omega}(z^2-\frac{2ai}{b}z-1)^2}dz$

取 $\omega=1$,0 是一级极点,$\frac{a-\sqrt{2}}{b}i$ 是二级极点

$$V_2=-\frac{2\pi}{b^2}\left[\frac{\sqrt{2}}{3}\beta_2+1\right]\cos t_0 \tag{18}$$

取 $\omega=2$,0 是一级极点,$\frac{a-\sqrt{2}}{b}i$ 是二级极点.

$$V_2=\frac{2\pi(\sqrt{2}a^2+4a+2\sqrt{2})\sin 2t_0}{b^3} \tag{19}$$

3) 当 $\omega=1$,

$$M(t_0)=-4\beta_0 b^2 V_1+2bFV_2=\frac{\sqrt{2}}{3}\left(\frac{4}{9}\beta_2^2-2\right)\beta_2\beta_0\pi-\frac{4\pi F\left(\frac{\sqrt{2}}{3}\beta_2+1\right)}{\sqrt{\frac{4}{9}\beta_2^2-2}}\cos t_0 \tag{20}$$

当 $\omega=2$,同样可得

$$M(t_0)=\frac{\sqrt{2}}{3}\left(\frac{4}{9}\beta_2^2-2\right)\beta_2\beta_0\pi-\frac{2\pi F(4\sqrt{2}\beta_2+24\beta_2+18\sqrt{2})}{2\beta_2^2-9}\sin t_0 \tag{21}$$

当 $\omega=1$ 时,由式(20)可得

$$4F\left(\frac{2}{3}\beta_2+1\right)>\frac{\sqrt{2}}{3}\left(\frac{4}{9}\beta_2^2-2\right)^{3/2}\beta_2\beta_0 \tag{22}$$

当 $\omega=2$ 时,由式(21)可得

$$2F(4\sqrt{2}\beta_2^2+24\beta_2+18\sqrt{2})>\frac{3\sqrt{2}}{2}\left(\frac{4}{9}\beta_2^2-2\right)^2\beta_2\beta_0 \tag{23}$$

就存在同宿点,系统可能发生混沌运动.

4.2 初始条件2

由方程(15)可取

$$x(t)=\frac{2}{a-b\cos t},$$

$$\frac{dx}{dt}=y(t)=\frac{-2\sin t}{(a-b\cos t)^2}$$

则 $M(t)=\int_{-\infty}^{+\infty}[-\beta_0 y^2(t)+Fy(t)\cos\omega(t+t_0)]dt$ 同法可得当 $\omega=1$,

$$M(t)=-\frac{\sqrt{2}}{3}\beta_0\beta_2(\frac{2}{3}\beta_2+\sqrt{2})(\frac{2}{3}\beta_2-\sqrt{2})\pi+2\sqrt{2}F\pi(\frac{2}{3}\beta_2-\sqrt{2})\sin t_0 \tag{24}$$

$$\text{当 } F>\frac{\beta_0}{6}\beta_2(\frac{2}{3}\beta_2+\sqrt{2}) \tag{25}$$

就存在同宿点,系统可能发生混沌运动.

当 $\omega=2$,

$$M(t_0)=-\frac{\sqrt{2}\beta_0}{3}\beta_2(\frac{2}{3}\beta_2+\sqrt{2})(\frac{2}{3}-\sqrt{2})+\frac{4\sqrt{2}F(\frac{2}{3}\beta_2-\sqrt{2})}{(\frac{2}{3}\beta_2+\sqrt{2})}\sin 2t_0 \tag{26}$$

则

$$F>\frac{\beta_0}{12}\beta_2(\frac{2}{3}\beta_2+\sqrt{2})^2 \tag{27}$$

就存在同宿点,系统可能发生混沌运动.

从式(22),式(23),式(25),式(27)可以明显看出,阻尼越小,激励越大,此动力系统越容易发生混沌运动.

在不同初始条件下,发生混沌运动的临界条件不同,从下面的相平面图(图1,图2)也可以看出发生的混沌运动.

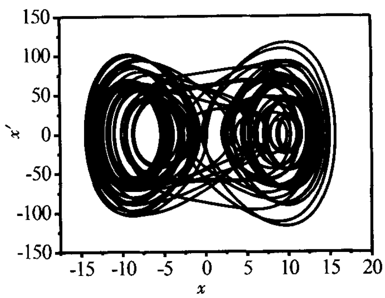

图1 相图 $\beta_0=0.1, F=1000$

Fig.1 Phase Plane $\beta_0=0.1, F=1000$

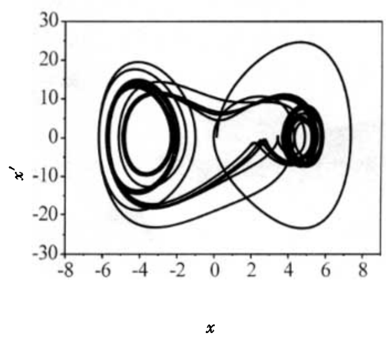

图2 相图 $\beta_0=0.5, F=100$

Fig.2 Phase Plane $\beta_0=0.5, F=100$

5 讨论

该文求出的自由振动方程(12)的准确解,使作者很容易求出 Melnikov 函数,对此类非线性动力系统在研究混沌现象可推广应用.对方程(14)与(15)的两个解无论取哪一个解,得到的 Melnikov 函数是一样的.

参 考 文 献

1 孙建恒,夏亨熹.网壳结构非线性动力稳定性分析.空间结构,1994,1(1):25~31(Sun Jianheng, Xia Hengxi. Nonlinear Stability Analysis of Space Frame Structures under Dynamic Loads. *Space Structure*,1994,1(1):25~31 (in Chinese))

2 周岱,刘红玉,李春祥.拉索-网壳结构的动力特性和非线性动力反应.振动与冲击,2002,21(1):7~11(Zhou Dai, Liu Hongyu, Li Chunxiang. Dynamic Feature and Nonlinear Dynamic Reaction of the Cable-stayed and Reticulated Shells Structure. *Vibration and Impact*,2002, 21(1):7~11(in Chiese))

3 桂国庆,林智斌.单层网壳的动力稳定性分析.南昌大学学报,2003,25(1):43~47(Gui Guoqing, Lin zhibin. Dynamic stability of single-layer reticulated spherical shells. *Journal of Nanchang University*,2003,25(1):43~47(in Chinese))

4 郭海山,沈世钊.单层网壳结构动力稳定性分析法.建筑结构学报,2003,24(3):1~9(Guo Haishan, Shen Shizhao. Analysis method of dynamic stability of single-layer reticulated domes. *Journal of Building Structures*, 2003,24(3):1~9(in Chinese))

5 Afraimovich VS, Glebsky I Ya, Nekorkin VI. Stability of stationary states and spatial choas in multidimensional lattice dynamical systems. *Random Comput Dynam*, 1994, 2,287~303

6 Bunimovich LA, CaHen EA. On the problem of Stability Dynamical Systems. *Journal of Differential Equations*, 1995,123:213~229

7 丁雪兴,何世权,王新志.静载荷下圆形三向网架的非线性振动.甘肃工业大学学报,2003,29(1):102~105 (Ding Xuexing, He Shiquan, Wang Xinzhi. Nonlinear Vibration of the Three-dimensional Circular Frame Subjected to Static load. *Journal of Gansu University of Technology*,2003,29(1):102~105(in Chinese))

8 王新志,王钢,赵艳影,叶开沅.圆形三向网架非线性动力稳定性分析.应用数学和力学,2004,25(4):331~336 (Wang Xinzhi, Wang Gang, Zhao Yanying, Ye Kaiyuan. The Nonlinear Dynamical stability analysis of the Circular Three-dimensional frame. *Applied Mathematics and Mechanics*,2004,25(4):331~336(in Chinese))

9 尹德钰,刘善维,钱若军.网壳结构设计.北京:中国建筑工业出版社,1996(Yin Deyu, Liu Shanwei, Qian Ruojun. Disign and Construction of Reticulated shells. Beijing: China Architecture & Building Press, 1996(in Chinese))

10 陈士华,陆君安.混沌动力学初步.武汉:武汉水利电力大学出版社,1998(Chen Shihua, Lu Junan. Elementary of Chaos Dynamics. Wuhan: Wuhan University of Water Conservancy and Electric Press, 1998(in Chinese))

NON-LINEAR DYNAMIC CHARACTERSISTICS OF SINGLE-LAYER SHALLOW CONICAL LATTICE SHELLS*

Wang Xinzhi　Liang Congxing　Li Lei　Han Mingjun　Ding Xuexing

(*School of Sciences, Lanzhou University of Technology, Lanzhou* 730050, *China*)

Abstract By using the simulated shells method, this paper established the axisymmetrical non-linear dynamic equations for three-dimensional single-layer shallow conical lattice shells with equilateral triangle mesh. Through the separating variables function method, a quadric and cubic non-linear differential equation was obtained by using the Galerkin method. In order to study chaos movement, a kind of non-linear free vibration differential equation was solved, the accurate solution of the single-layer shallow conical cattice shells was obtained, and the critical condition was obtained by using the Melnikov function. Moreover, the numerical-graphic method also confirmed the existence of chaos.

Key words single-layer lattice shells, separating variables, critical condition, chaos

Received 01 August 2004, revised 02 September 2004.

* The project supported by the Natural Science Foundation of Gansu Province China(ZS021-A25-007-Z)