圆柱壳在热载荷及微扰外压作用下的分岔*

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摘要 在考虑温度对圆柱壳材料性能影响的基础上,建立了圆柱壳在扰动外压作用下的几何非线性动力控制方程.并采用伽辽金原理及 Melnikov 法研究了圆柱壳在热载荷及微扰外压作用下的分岔,进一步讨论分析了温度、Batdorf 参数等因素对圆柱壳发生混沌运动区域的影响,得出了随温度、Batdorf 参数的增大,混沌运动区域将越来越大的结论.

关键词 圆柱壳,热载荷,分岔,混沌

引 言

圆柱壳作为一种工程结构在航天航空、机械、石油化工、核电站安全壳等工程中得到了广泛应用.在这些工程实际中圆柱壳经常受到热载荷及扰动外压的作用,随着温度升高热载荷会对圆柱荷结构产生很大的危害,有时可能造成很大的经济损失.所以温度分布所产生的热强度问题已成为实际工程中的重大问题,而与结构寿命有关的热应力分析在设计中已占据着重要地位,人们越来越关注温度效应对板壳振动的影响.本文在考虑温度对圆柱壳材料性能影响的基础上,讨论分析了大挠度圆柱壳在周期微扰动外压作用下的分岔,在考虑几何非线性的同时,也计及了温度效应,利用伽辽金原理及 Melnikov 方法确定动力系统出现马蹄形时参数应满足的条件,并讨论分析了温度、Batdorf 参数等因素对圆柱壳发生混沌运动区域的影响.

1 温度对壳材料性能影响

当圆柱壳的工作环境温度较高时,对圆柱壳的动力特性有一定影响.圆柱壳受温度影响的原因主要有以下两个方面:

a) 圆柱壳材质具有随温度变化而膨胀或收缩的特性.

b) 圆柱壳材料弹性模量随温度变化的特性.假设圆柱壳在均匀温度场中工作,由于物质的宏观性呈连续性变化,根据热弹性理论得[1,5]

$$\begin{cases} E = E_0 + E_1 T_0 + E_2 T_0^2 & (1) \\ \alpha_s = \alpha_{s0} + \alpha_{s1} T_0 + \alpha_{s2} T_0^2 & (2) \end{cases}$$

上式表明弹性模量 E、热膨胀系数 α_s 随温度呈非线性变化.考虑到圆柱壳的工作环境温度,本文忽略掉二次项即假设温度对圆柱壳材料的弹性模量及热膨胀系数的影响呈线性变化

$$\begin{cases} E = E_0 + (1 + \alpha_E T_0) & (3) \\ \alpha_s = \alpha_{s0}(1 + \alpha_k T_0) & (4) \end{cases}$$

2 基本方程

根据经典壳体理论且计及热效应,圆柱壳在扰动外压作用下几何非线性动力控制方程为

$$\begin{cases} D(1 + \alpha_E T_0)\nabla^4 W + \nabla^2 M_T + \gamma \dfrac{\partial W}{\partial t} - \dfrac{1}{R}\dfrac{\partial^2 F}{\partial x^2} + \rho h \dfrac{\partial^2 W}{\partial t^2} = L(W,F) + q(x,y,t) & (5) \\ \dfrac{1}{E_0 h(1 + \alpha_E T_0)}\nabla^4 F + \dfrac{1-\mu}{E_0 h(1 + \alpha_E T_0)}\nabla^2 N_T + \dfrac{1}{R}\dfrac{\partial^2 W}{\partial x^2} = -\dfrac{1}{2}L(w,w) & (6) \\ N_x = \dfrac{\partial^2 F}{\partial y^2}, N_y = \dfrac{\partial^2 F}{\partial x^2}, N_{xy} = \dfrac{\partial^2 F}{\partial x \partial y} & (7) \end{cases}$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

其中 $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

2003-09-14 收到第一稿,2003-10-15 收到修改稿.
* 湖南省自然科学基金资助项目(00JJY20114).

$$L(\ )=\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}-2\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y}+\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2}$$

面内位移 U,V 与 W,F 的关系为

$$\begin{cases}\frac{\partial U}{\partial x}+\frac{1}{2}(\frac{\partial w}{\partial x})^2=\frac{1}{E_0h(1+\alpha_ET_0)}(\frac{\partial^2F}{\partial y^2}-\mu\frac{\partial^2F}{\partial x^2}) & (8)\\ \frac{\partial V}{\partial y}-\frac{w}{R}+\frac{1}{2}(\frac{\partial w}{\partial y})^2=\frac{1}{E_0h(1+\alpha_ET_0)}(\frac{\partial^2F}{\partial x^2}-\mu\frac{\partial^2F}{\partial y^2}) & (9)\\ \frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}=-\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}-\frac{2(1+\mu)}{E_0h(1+\alpha_ET_0)}\frac{\partial^2F}{\partial x\partial y} & (10)\end{cases}$$

在以上各式中,w 为壳的位移,F 为应力函数,$D=\frac{E_0h^3}{12(1-\mu^2)}$为弯曲刚度,$\mu$ 为泊松比,ρ 为密度,h 为壳厚,R 为圆柱壳半径,N_T,M_T 为热力、热矩,$q(x,y,t)$为扰动外压.

热力、热矩的表达式为

$$(N_T,M_T)=\frac{E_0\alpha_{s0}(1+\alpha_ET_0)(1+\alpha_kT_0)}{1-\mu}\cdot\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,Z)T_0\mathrm{d}z \quad (11)$$

由(8)式可得圆柱壳端部伸长为

$$\triangle_x=-\frac{1}{2\pi R}\int_0^l\int_0^{2\pi R}\left[\frac{1}{E_0h(1+\alpha_ET_0)}(\frac{\partial^2F}{\partial y^2}-\mu\frac{\partial^2F}{\partial x^2})-\frac{1}{2}(\frac{\partial w^2}{\partial x})+\frac{1-\mu}{E_0h(1+\alpha_ET_0)}N_T\right]\mathrm{d}x\mathrm{d}y \quad (12)$$

由(9)式可得圆柱壳闭合条件(或周期性条件)为

$$\int_0^{2\pi R}\left[\frac{1}{E_0h(1+\alpha_ET_0)}(\frac{\partial^2F}{\partial x^2}-\mu\frac{\partial^2F}{\partial y^2})-\frac{1}{2}(\frac{\partial w}{\partial y})^2+\frac{w}{R}+\frac{1-\mu}{E_0h(1+\alpha_ET_0)}N_T\right]\mathrm{d}y=0 \quad (13)$$

假定圆柱壳两端为不可移简支,其边界条件为

$$x=0,l,\quad w=\frac{\partial^2w}{\partial x^2}=0,\quad \triangle_x=0 \quad (14)$$

设(5)式的解为

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varnothing(t)\sin\frac{m\pi x}{l}\sin\frac{\lambda\pi y}{l},\quad\left(\lambda=\frac{nl}{\pi R}\right) \quad (15)$$

设周期扰动外压力为

$$q(x,y,t)=(p_0+p_1\cos\omega_0t)\cdot\sin\frac{m\pi x}{l}\sin\frac{\lambda\pi y}{l} \quad (16)$$

把(15)式代入(6)式中可求得

$$F(x,y,t)=\frac{m^2E_0h(1+\alpha_ET_0)l^2\varnothing}{\pi^2(m^2+\lambda^2)R}\cdot\sin\frac{m\pi x}{l}\sin\frac{\lambda\pi y}{l}+\frac{E_0h(1+\alpha_ET_0)\varnothing^2}{32}\cdot\left(\frac{\lambda^2}{m^2}\cos\frac{2m\pi x}{l}+\frac{m^2}{\lambda^2}\cos\frac{2\lambda\pi y}{l}\right)+\frac{1}{2}p_xy^2+\frac{1}{2}p_yx^2+\frac{1}{2}p_{xy}xy \quad (17)$$

把有关各式代入(5)式中,并利用伽辽金原理且无量纲化为

$$\ddot{\varphi}-\varphi+\varphi^3=\epsilon(f_0+f_1\cos\Omega\tau-c\dot{\varphi}) \quad (18)$$

式中

$$\alpha=E_0(1+\alpha_ET_0)\left[\frac{\pi^2\alpha_{s0}T_0(1+\alpha_kT_0)(\lambda^2+m^2)}{(1-\mu)\rho\eta\xi^2R^3}-\frac{\pi^4(m^2+\lambda^2)^2}{12\rho R^2K^2}-\frac{m^4}{\rho R^2(m^2+\lambda^2)^2}\right]$$

$$\beta=\frac{\pi^4E_0(1+\alpha_ET_0)(m^4+\lambda^4)}{16\rho R^4\xi^4}+\frac{\pi^4E_0(1+\alpha_ET_0)(\lambda^4+2\mu\lambda^2m^2+m^4)}{8(1-\mu^2)\rho R^4\xi^4}$$

$$\varnothing=\varphi\sqrt{\frac{\alpha}{\beta}},t=\frac{\tau}{\sqrt{\alpha}},\Omega=\frac{\omega_0}{\sqrt{\alpha}},$$

$$\eta=\frac{h}{R},\xi=\frac{l}{R}$$

$$c=\frac{V}{\epsilon\rho h\alpha^{\frac{1}{2}}},f_0=\frac{\sqrt{\beta}P_0}{\epsilon\rho h\alpha^{\frac{3}{2}}},$$

$$f_1=\frac{\sqrt{\beta}P_1}{\epsilon\rho h\alpha^{\frac{3}{2}}},K=\frac{l^2\sqrt{1-\mu^2}}{Rh}$$

3 圆柱壳热分岔

若无周期扰动外压时 $\epsilon=0$,则动力系统是 Hamilton 系统,它的三个不动点中,(0,0)是中心,(±1,0)是两个平衡点. 所以,两个平衡点处的 Hamilton 能量函数为

$$H=\frac{1}{2}\dot{\varphi}^2-\frac{1}{2}\varphi^2+\frac{1}{4}\varphi^4 \quad (19)$$

如初始条件 $\varphi(0)=0$,可得平衡点(±1,0)对应的同宿轨道为

$$\begin{cases}\varphi(\tau)=\pm\sqrt{2}\,\mathrm{sech}\tau\\ \dot{\varphi}(\tau)=\mp\sqrt{2}\,\mathrm{th}\tau\,\mathrm{sech}\tau\end{cases} \quad (20)$$

根据 Melnikov 法同宿轨道的存在是出现分岔的标志. 同宿轨道破裂将导致系统出现复杂的分岔行为,可能出现混沌运动.

把(18)式化为

$$\dot{\xi} = g_0(\xi) + \varepsilon g_1(\xi) \tag{21}$$

其中

$$\xi = \begin{Bmatrix} \varphi(\tau) \\ \dot{\varphi}(\tau) \end{Bmatrix}, g_0(\xi) = \begin{Bmatrix} \dot{\varphi} \\ \varphi(1-\varphi^2) \end{Bmatrix},$$

$$g_1(\xi) = \begin{Bmatrix} 0 \\ f_0 + f_1\cos\Omega\tau - \eta\dot{\varphi} \end{Bmatrix}$$

系统受扰动后,异宿环破裂,用 Melnikov 函数来测量两轨道间的距离

$$M(\tau_0) = \int_{-\infty}^{\infty} g_0[\xi(\tau)] \wedge g_1[\xi(\tau),(\tau+\tau_0)]\mathrm{d}\tau =$$

$$-\frac{4}{3}c \pm 2\sqrt{2}f_0 \pm \sqrt{2}f_1\pi\Omega\operatorname{sech}\frac{\pi\Omega}{2}\sin\Omega\tau_0 \tag{22}$$

令 $M(\tau_0)=0$ 可得发生混沌运动临界条件为

a)当$\dfrac{f_0}{f_1} < \dfrac{\pi\Omega/2}{\operatorname{ch}\frac{\pi\Omega}{2}}$时

$$\frac{2}{3\sqrt{2}\left(\dfrac{\pi\Omega/2}{\operatorname{ch}\frac{\pi\Omega}{2}} - \dfrac{f_0}{f_1}\right)} < \frac{f_1}{c} \tag{23}$$

b)当$\dfrac{f_0}{f_1} > \dfrac{\pi\Omega/2}{\operatorname{ch}\frac{\pi\Omega}{2}}$时

$$\frac{2}{3\sqrt{2}\left(\dfrac{\pi\Omega/2}{\operatorname{ch}\frac{\pi\Omega}{2}} + \dfrac{f_0}{f_1}\right)} < \frac{f_1}{c} < \frac{2}{3\sqrt{2}\left(\dfrac{f_0}{f_1} - \dfrac{\pi\Omega/2}{\operatorname{ch}\frac{\pi\Omega}{2}}\right)} \tag{24}$$

4 实例计算及讨论

为了讨论分析温度、Batdorf 参数等因素对圆柱壳发生混沌运动区域的影响,取壳的参数为:

$$l = 1\ \mathrm{m}, E_0 = 2.1\times10^{11}\ \mathrm{N/m^2},$$

$$\rho = 7.8\times10^3\ \mathrm{kg/m^3},$$

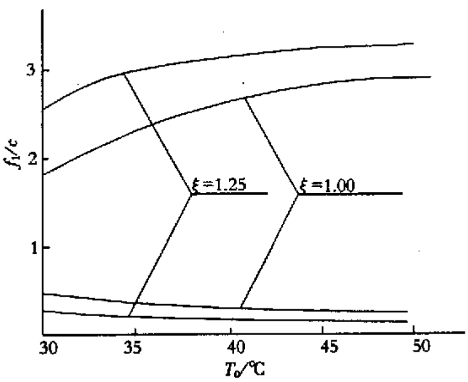
$$\alpha_E = -3.95\times10^{-4}\ ^\circ\mathrm{C}, \mu = 0.3,$$

$$\alpha_{s0} = 1.2\times10^{-5}\ ^\circ\mathrm{C}, \alpha_k = 2.26\times10^{-9}\ ^\circ\mathrm{C},$$

$$\omega_0 = 7.5\times10^3\ \mathrm{rad/s}, m = n = 1.$$

对图 1,图 2,图 3 进行分析可知:

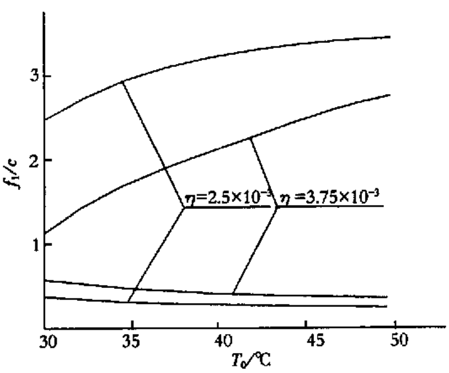
i)随着温度升高,混沌运动区域将越来越大,这说明混沌运动区域对温度变化很敏感. 但温度变化引起壳材料弹性模量及热膨胀系数的改变对混沌运动区域的影响不大,最大误差仅在 0.1%左右在图中无法反映出来,可以忽略不计.



($\frac{f_0}{f_1}=0.8, \eta=2.5\times10^{-3}$)

图 1 混沌运动区域 I

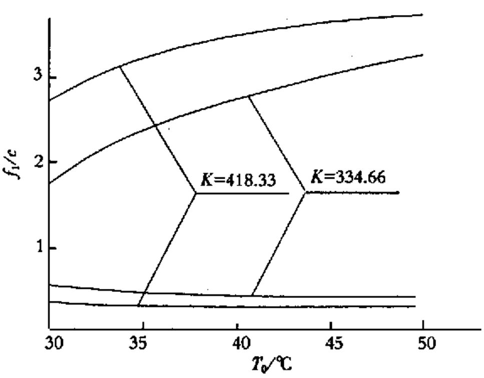
Fig. 1 Chaotic motion region I



($\frac{f_0}{f_1}=0.8, \xi=1.25$)

图 2 混沌运动区域 II

Fig. 2 Chaotic motion region II



($\frac{f_0}{f_1}=0.8, \eta=2.5\times10^{-3}$)

图 3 混沌运动区域 III

Fig. 3 Chaotic motion region III

ii)当 Batdorf 参数 K 增大时即长径比$\frac{l}{R}$、长厚比$\frac{l}{h}$增加，混沌运动区域将越来越大；当厚径比$\frac{h}{R}$增大时，混沌运动区域则越来越小.

iii)在热状态下混沌运动区域受厚径比$\frac{h}{R}$的影响要比受 Batdorf 参数即长径比$\frac{l}{R}$、长厚比$\frac{l}{h}$的影响要大，这说明混沌运动区域受厚径比的变化影响最大、最敏感.

5 结　论

本文在考虑温度对圆柱壳材料性能影响的基础上，建立了圆柱壳在扰动外压作用下的几何非线性动力控制方程；然后，采用伽辽金原理及 Melnikov 法研究了圆柱壳在热载荷及微扰外压作用下的分岔. 通过具体计算及分析，可以得到随着温度的升高、Batdorf 参数的增大，圆柱壳的混沌运动区域将越来越大的结论. 该结论可供工程设计人员借鉴和参考.

参　考　文　献

1 王洪纲主编. 热应力. 昆明：云南科技出版社，1996：73～77 (Wang Honggang. Thermal Stress. Kunming: Science and Technology Press, 1996: 73～77(in chinese))

2 Yang XL, Sathna PR. Local and global difurcations in parametrically excited vibrations of nearly square plates. *Journal of applied mechanics*, 1991, 26: 190～197

3 Abhyankar NS, Hall Ek, Hangud SV. Chaotic Vibrations of beam: Numerical solution of partial differtial equations. *J Applied Mech*, 1993, 60(3): 167～174

4 Tang DM, Dowell EH. On the threshold force for chaotic motion of a forced buckled beam. *J Applied Mech*, 1988, 55(2): 190～196

5 王洪纲. 热弹性力学概论. 北京：清华大学出版社，1989 (Wang Hong-gang Introduction of Thermal Elastic Mechanics. Beijing: Tsinghua University Press, 1989(in Chinese))

6 龙运佳. 混沌运动研究方法与实践. 北京：清华大学出版社，1997 (Long Yunjia. The Research Method and Practice of Chaotic Motion. Beijing: Tsinghua University Press, 1997(in Chinese))

7 李骊. 强非线性振动系统的定性理论与定量方法. 北京：科学出版社，1997 (Li Li, The Qualitative Theory and Quantitative Method of Strong Nonlinear Vibration Systems. Beijing: Science Press, 1997(in Chineses))

BIFURCATION OF CYLINDRICAL SHELLS UNDER THERMAL LOAD AND DISTURBING EXTERNAL PRESSURE*

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Abstract Based on the effects of temperature upon the material properties of cylindrical shells, this paper established a geometrical nonlinear dynamic control equation of cylindrical shells under disturbing external pressure, and studied the bifurcation of cylindrical shells under thermal load and disturbing external pressure by using Galerkin's principle and Melnikov's method. It also discussed the effects of temperature, Batdorf's parameter etc. upon the chaotic motion region of cylindrical shells. The results show that the chaotic motion region enlarges when the temperature or the Batdorf's parameter increases.

Key words cylindrical shell, thermal load, bifurcation, chaos

Received 14 September 2003, revised 20 November 2003.

* The project supported by the Natural Science Foundation of Hunan Province, China (00JJY20114).