非线性边界条件下弯扭耦合梁方程组的吸引子 *

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摘要　研究具有实际背景的弯曲与扭转联合作用下梁方程组在非线性边界条件下的吸引子.首先通过Faedo-Galerkin方法证明整体解的存在唯一性,其次证明系统存在有界吸收集和半群的渐近光滑性,最后得到全局吸引子的存在.

关键词　弯曲与扭转联合，　非线性边界条件，　整体解，　全局吸引子

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引言

对于力学中梁结构所确定的无穷维动力系统的研究,很多学者讨论的都是假设梁在对称平面内的弯曲振动,如果不是这种情况,通常梁的弯曲振动将会与扭转振动结合起来.对梁的弯曲与扭转联合作用的数学模型最初由学者Timoshenko[1]提出：

$$\begin{cases} EI\dfrac{\partial^4 w}{\partial x^4} = -\rho\hat{A}\dfrac{\partial^2}{\partial t^2}(w - c\varphi) + N\dfrac{\partial^2 w}{\partial x^2}, \\ R\dfrac{\partial^2 \varphi}{\partial x^2} - R_1\dfrac{\partial^4 \varphi}{\partial x^4} = -\rho\hat{A}c\dfrac{\partial^2}{\partial t^2}(w - c\varphi) + \rho I_1\dfrac{\partial^2 w}{\partial t^2}. \end{cases}$$

其中$w, E, I, \hat{A}, N, R, R_1, \varphi, \rho, c, I_1$分别表示挠度、杨氏模量、截面惯性矩、梁的横截面面积、轴向内力、均匀扭转的扭转刚度、翘曲刚度、翘曲函数、单位梁长的密度、阻尼系数、横截面的形心极惯性矩.

根据系统影响因素的不同,国内外许多学者分别从不同的角度进一步建立了与振动因素相对应的数学模型.Timoshenko[2]最先提出了该模型,并证明解的存在唯一性；Andrade[3]在Timoshenko的基础上增加一非线性项,在上述相同初始条件下证明了解的存在性、唯一性及正则性；张建文等[4,5]研究了在含有二阶空间导数的线性边界条件下的演化方程的整体解的存在性、唯一性.

以上多数研究的是该类耦合梁方程组在线性边界条件下解的存在唯一性,而对于非线性边界下解的存在唯一性仅有少数研究,如李银玉[6]探究了在含有三阶空间导数的非线性边界下弱解的存在性问题.但是关于弯曲与扭转联合作用下梁方程组系统的吸引子未见相关报道,因此本文主要探究如下耦合梁方程组

$$\begin{cases} u_{tt} + cv_{tt} + \eta_1 u_t + \alpha u_{xxxx} - (a + b\|u_x\|^2)u_{xx} = 0, \\ cu_{tt} + \gamma v_{tt} + \eta_2 v_t + \delta v_{xxxx} - \beta_0 v_{xx} = 0. \end{cases} \tag{1}$$

其中，$c, \eta_1, \alpha, b, \gamma, a, \eta_2, \delta, \beta_0$为常数，且$c^2 < \gamma, x \in (0, l)$.在含有三阶空间导数的非线性边界条件

$$\begin{cases} u(0,t) = u_x(l,t) = u_{xx}(0,t) = 0, \\ u_{xxx}(l,t) = f_1(u(l,t)) + g_1(u_t(l,t)), \\ v(0,t) = v_x(l,t) = v_{xx}(0,t) = 0, \\ v_{xxx}(l,t) = f_2(v(l,t)) + g_2(v_t(l,t)) \end{cases} \tag{2}$$

和初始条件

$$\begin{cases} u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x), \\ v(x,0) = v^0(x), \quad v_t(x,0) = v^1(x). \end{cases} \tag{3}$$

下的全局吸引子.

在本文中,系统(1)–系统(3)描述了一个弯扭耦合梁的振动,条件(2)的第一、三方程表示该梁在左端点$x = 0$处被铰接.第二、四方程与$x = l$处的剪切力有关且依赖于函数f_1, g_1, f_2, g_2.进一步作为物理解释,梁的右侧受到非线性弹性力,用f_1, f_2表示,并受到函数g_1, g_2表示的一个非线性阻尼.

1　空间和函数假设

假设梁方程组定义在一维空间,令$\Omega = (0, l)$,

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本文分析基于如下Sobolev空间

$U=\{u\in H^1(0,l)|u(0)=0\}$,

$V=\{u\in H^2(0,l)|u(0)=u_x(l)=0\}$,

$W=\{u\in V\cap H^4(0,l)|u_{xx}(0)=0\}$.

设空间$H_1=W\times W\times W\times W$,由边界条件(2)可知初始条件满足下式

$$\begin{cases}u_{0xxx}(l)=f_1(u_0(l))+g_1(u_1(l)),\\ v_{0xxx}(l)=f_2(v_0(l))+g_2(v_1(l))\end{cases}\tag{4}$$

设空间$H_0=V\times U\times V\times U=\bar{H}_1$,在$H_0$定义如下范数:$\|(u,u_t,v,v_t)\|_{H_0}^2=\|u_{xx}\|^2+\|u_t\|^2+\|v_{xx}\|^2+\|v_t\|^2$.

下面对函数作出假设.函数$f_i,g_i:R\to R\in C^1(R)$并且满足$f_i(0)=g_i(0)$,存在常数$k_i,p_i,m_i,\rho,r\geq 0$ $(i=1,2)$,对$\forall u,v\in R$有

$$f_i(u)u\geq 0,\quad f_i(u)u-2\hat{f}_i(u)\geq 0\tag{5}$$

$$|f_i(u)-f_i(v)|\leq k_i(1+|u|^\rho+|v|^\rho)|u-v|\tag{6}$$

$$(g_i(u)-g_i(v))(u-v)\geq p_i|u-v|^2\tag{7}$$

$$|g_i(u)-g_i(v)|\leq m_i(1+|u|^r+|v|^r)|u-v|\tag{8}$$

其中$\hat{f}_i(z)=\int_0^z f(s)\mathrm{d}s$,在本文中均取$\rho=r=0$.

2 整体解的存在唯一性

定理2.1 设空间H_1和函数$f_i(\cdot),g_i(\cdot)$的假设条件均成立,若对于任何初值$(u^0,u^1,v^0,v^1)\in H_1$,则系统(1)-(3)有唯一的正则解$(u(x,t),v(x,t))$满足:

$u\in L_{loc}^\infty(R^+,W)\cap C^0([0,\infty);\ V)\cap C^1([0,\infty);\ U)$,

$v\in L_{loc}^\infty(R^+,W)\cap C^0([0,\infty);\ V)\cap C^1([0,\infty);\ U)$,

$\|u_t\|^2+\|v_t\|^2+\|u_{xx}\|^2+\|v_{xx}\|^2+\|u_x\|^2+\|v_x\|^2\leq M_1$.

其中M_1是一个正常数.

证明:(近似解) 设ω_i为W的基,对$m\in N$,令$W_m=\{\omega_1,\omega_2,\cdots,\omega_m\}$,则有函数

$$u^m(t)=\sum_{i=1}^m g_{im}(t)w_i,\ \ v^m(t)=\sum_{i=1}^m h_{im}(t)w_i,$$

对$\forall\omega\in W_m$,此函数对$\{u^m(t),v^m(t)\}$是如下逼近系统的解:

$$\begin{aligned}&(u_{tt}^m(t),\omega)+c(v_{tt}^m(t),\omega)+\eta_1(u_t^m(t),\omega)+\\&(a+b\|u_x^m(t)\|^2)(u_x^m(t),\omega_x)+\alpha(u_{xx}^m(t),\omega_{xx})+\\&\alpha f_1(u^m(l,t))\omega(l)+\alpha g_1(u_t^m(l,t))\omega(l)=0\end{aligned}\tag{9}$$

$$\begin{aligned}&c(u_{tt}^m(t),\omega)+\gamma(v_{tt}^m(t),\omega)+\eta_2(v_t^m(t),\omega)+\\&\delta(v_{xx}^m(t),\omega_{xx})+\beta_0(v_x^m,\omega_x)+\\&\delta f_2(v^m(l,t))\omega(l)+\delta g_2(v_t^m(l,t))\omega(l)=0\end{aligned}\tag{10}$$

$$\begin{aligned}&u^m(0)=u^{m0}(x),\quad u_t^m(0)=u^{m1}(x),\\&v^m(0)=v^{m0}(x),\quad v_t^m(0)=v^{m1}(x)\end{aligned}\tag{11}$$

且初始条件满足

$$\begin{cases}u^m(0)=u^{m0}\to u^0,\quad u_t^m(0)=u^{m1}(x)\to u^1,\\ v^m(0)=v^{m0}\to v^0,\quad v_t^m(0)=v^{m1}(x)\to v^1.\end{cases}$$

事实上,上述方程组是关于时间t的$m\times m$常微分方程组.由Peano定理知,在$[0,t_m]$上存在一个解$(u^m(x,t),v^m(x,t))$,接下来对近似解估计,把区间$[0,t_m]$延拓到$[0,T]$,$\forall T>0$.

估计1:在式(9)中取$\omega=2u_t^m(t)$,在式(10)中取$\omega=2v_t^m(t)$,两式相加后在区间$(0,t)(t\leq t_m)$上积分,得

$$\begin{aligned}&\|u_t^m(t)\|^2+\alpha\|u_{xx}^m(t)\|^2+a\|u_x^m(t)\|^2+\frac{b}{2}\|u_x^m(t)\|^4+\\&\gamma\|v_t^m(t)\|^2+\delta\|v_{xx}^m(t)\|^2+\beta_0\|v_x^m(t)\|^2+\\&2\alpha\hat{f}_1(u^m(l,t))+2\delta\hat{f}_2(v^m(l,t))+\\&2\alpha\int_0^t g_1(u_t^m(l,t))u_t^m(l,t)\mathrm{d}t+2\delta\int_0^t g_2(v_t^m(l,t))v_t^m(l,t)\mathrm{d}t\\&=\|u_t^m(0)\|^2+\alpha\|u_{xx}^m(0)\|^2+a\|u_x^m(0)\|^2+\\&2c(u_t^m(0),v_t^m(0))+\frac{b}{2}\|u_x^m(0)\|^4+\gamma\|v_t^m(0)\|^2+\\&\delta\|v_{xx}^m(0)\|^2+\beta_0\|v_x^m(0)\|^2+\\&2\alpha\hat{f}_1(u^m(l,0))\ +2\delta\hat{f}_2(v^m(l,0))\ -2c(u_t^m(t),v_t^m(t))-\\&2\eta_1\int_0^t\|u_t^m(t)\|^2\mathrm{d}t-2\eta_2\int_0^t\|v_t^m(t)\|^2\mathrm{d}t.\end{aligned}\tag{12}$$

由函数假设条件(5)和条件(7)知:等式左端后四项都大于零.因为$c^2<\gamma$,所以$\frac{c}{\sqrt{\gamma}}<1$,利用Young不等式得

$$-2c\|u_t^m(t)\|\cdot\|v_t^m(t)\|\geq-\frac{c}{\sqrt{\gamma}}\|u_t^m(t)\|^2-c\sqrt{\gamma}\|v_t^m(t)\|^2.$$

则式(12)变为下面不等式

$$\begin{aligned}&(1-\frac{c}{\sqrt{\gamma}})\|u_t^m(t)\|^2+(\gamma-c\sqrt{\gamma})\|v_t^m(t)\|^2+\alpha\|u_{xx}^m(t)\|^2+\\&\delta\|v_{xx}^m(t)\|^2+a\|u_x^m(t)\|^2+\frac{b}{2}\|u_x^m(t)\|^4+\beta_0\|v_x^m(t)\|^2\\&\leq 2\eta_1\int_0^t\|u_t^m(t)\|^2\mathrm{d}t+2\eta_2\int_0^t\|v_t^m(t)\|^2\mathrm{d}t+C.\end{aligned}$$

其中C是一个正常数,在本文中不同地方的C是不同的常数.利用Growall引理,得

$\|u_t^m(t)\|,\|v_t^m(t)\|,\|u_{xx}^m(t)\|,\|v_{xx}^m(t)\|,$

$\|u_x^m(t)\|,\ \|v_x^m(t)\|,\|u^m(t)\|,\ \|v^m(t)\|\leq M_1.$

其中M_1是一个与m,t无关的正常数.

估计2:在式(9)中取$\omega=u_{tt}^m(0)$,在式(10)中取$\omega=v_{tt}^m(0)$,并且$t=0$,变分后两式相加后得

$$\begin{aligned}&\|u_{tt}^m(0)\|^2+\gamma\|v_{tt}^m(0)\|^2+2c(u_{tt}^m(0),v_{tt}^m(0))\\&=(a+b\|u_x^m(0)\|^2)(u_{xx}^m(0),u_{tt}^m(0))-\eta_1(u_t^m(0),u_{tt}^m(0))-\\&\eta_2(v_t^m(0),v_{tt}^m(0))-\alpha(u_{xxxx}^m(0),u_{tt}^m(0))\ -\\&\beta_0(v_{xx}^m(0),v_{tt}^m(0))-\delta(v_{xxxx}^m(0),v_{tt}^m(0)).\end{aligned}$$

利用Cauchy不等式，由初始条件收敛性，得

$$\|u_{tt}^m(0)\|^2+\gamma\|v_{tt}^m(0)\|^2+2c(u_{tt}^m(0),v_{tt}^m(0))\leqslant C\|u_{tt}^m(0)\|+C\|v_{tt}^m(0)\|.$$

类似于估计1中的讨论，进而根据配方法可得

$$\|u_{tt}^m(0)\|\leqslant M_2,\qquad \|v_{tt}^m(0)\|\leqslant M_2,\quad \forall t\in[0,T].$$

估计3：在式(9)中分别取$t=t+\xi$和$t=t$，得新的两式，然后将两式作差后再取$\omega=u_t^m(t+\xi)-u_t^m(t)$. 同理，在式(10)中用类似方法，但此时取$\omega=v_t^m(t+\xi)-v_t^m(t)$，最后两式相加得

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\phi^m(t,\xi)+\eta_1\|u_t^m(t+\xi)-u_t^m(t)\|^2+\\&\quad\eta_2\|v_t^m(t+\xi)-v_t^m(t)\|^2\\&=b\|u_x^m(t+\xi)\|^2(u_x^m(t+\xi),u_{xt}^m(t+\xi)-u_{xt}^m(t))-\\&\quad b\|u_x^m(t)\|^2(u_x^m(t),u_{xt}^m(t+\xi)-u_{xt}^m(t))-\\&\quad\alpha[f_1(u^m(l,t+\xi))-f_1(u^m(l,t))](u_t^m(l,t+\xi)-u_t^m(l,t))-\\&\quad\alpha[g_1(u_t^m(l,t+\xi))-g_1(u_t^m(l,t))](u_t^m(l,t+\xi)-u_t^m(l,t))-\\&\quad\delta[f_2(v^m(l,t+\xi))-f_2(v^m(l,t))](v_t^m(l,t+\xi)-v_t^m(l,t))-\\&\quad\delta[g_2(v_t^m(l,t+\xi))-g_2(v_t^m(l,t))](v_t^m(l,t+\xi)-v_t^m(l,t))\end{aligned}\tag{13}$$

其中

$$\begin{aligned}\phi^m(t,\xi)&=\|u_t^m(t+\xi)-u_t^m(t)\|^2+a\|u_x^m(t+\xi)-u_x^m(t)\|^2+\\&\quad\alpha\|u_{xx}^m(t+\xi)-u_{xx}^m(t)\|^2+\gamma\|v_t^m(t+\xi)-v_t^m(t)\|^2+\\&\quad\beta_0\|v_x^m(t+\xi)-v_x^m(t)\|^2+\delta\|v_{xx}^m(t+\xi)-v_{xx}^m(t)\|^2+\\&\quad 2c(u_t^m(t+\xi)-u_t^m(t),v_t^m(t+\xi)-v_t^m(t)).\end{aligned}$$

因为$u(0,t)=u_x(l,t)=u_{xx}(0,t)=0$[7]，所以有

$$\|u\|_\infty\leqslant\sqrt{l}\,\|u_x\|,\ \|u_x\|_\infty\leqslant\sqrt{l}\,\|u_{xx}\|,\quad \|u_x\|\leqslant l\|u_{xx}\|\tag{14}$$

由式(14)、中值定理、函数假设条件(7)对式(13)右端估计、再利用Growall不等式得

$$\phi^m(t,\xi)\leqslant\phi^m(0,\xi)\exp(CT),\qquad\forall t\in[0,T].$$

上述不等式两边同时除以ξ^2，并令$\xi\to0$，根据导数的定义得

$$\begin{aligned}&\|u_{tt}^m(t)\|^2+a\|u_{xt}^m(t)\|^2+\alpha\|u_{xxt}^m(t)\|^2+\gamma\|v_{tt}^m(t)\|^2+\\&\quad\beta_0\|v_{xt}^m(t)\|^2+\delta\|v_{xxt}^m(t)\|^2+2c(u_{tt}^m(t),v_{tt}^m(t))\\&\leqslant[\|u_{tt}^m(0)\|^2+a\|u_{xt}^m(0)\|^2+\alpha\|u_{xxt}^m(0)\|^2+\\&\quad\gamma\|v_{tt}^m(0)\|^2+\beta_0\|v_{xt}^m(0)\|^2+\delta\|v_{xxt}^m(0)\|^2+\\&\quad 2c(u_{tt}^m(0),v_{tt}^m(0))]\exp(CT)\end{aligned}\tag{15}$$

由估计2和初始条件的收敛性可知：存在仅依赖于T的常数$M_3>0$，对$\forall t\in[0,T]$有

$$\|u_{tt}^m(t)\|,\|u_{xt}^m(t)\|,\|u_{xxt}^m(t)\|,\|v_{tt}^m(t)\|,\|v_{xt}^m(t)\|,\|v_{xxt}^m(t)\|\leqslant M_3.$$

唯一性：假设(u,v)，$(\bar u,\bar v)$是方程组的两组解，记$w=u-\bar u$，$\bar w=v-\bar v$，则$w,\bar w$满足系统

$$\begin{cases}w_{tt}+c\bar w_{tt}+\eta_1w_t+\alpha w_{xxxx}-aw_{xx}\\\quad=b(\|u_x\|^2-\|\bar u_x\|^2)\bar u_{xx}+b\|u_x\|^2w_{xx},\\cw_{tt}+\gamma\bar w_{tt}+\eta_2\bar w_t+\delta\bar w_{xxxx}-\beta_0\bar w_{xx}=0\end{cases}\tag{16}$$

和初始条件

$$w(0)=\bar w(0)=0,\qquad w_t(0)=\bar w_t(0)=0\tag{17}$$

及边界条件

$$\begin{cases}w(0,t)=w_x(l,t)=w_{xx}(0,t)=0,\\\bar w(0,t)=\bar w_x(l,t)=\bar w_{xx}(0,t)=0,\\w_{xxx}(l,t)=f_1(u(l,t))-f_1(\bar u(l,t))+\\\qquad g_1(u_t(l,t))-g_1(\bar u_t(l,t))\overset{\Delta}{=}\Lambda f+\Lambda g,\\\bar w_{xxx}(l,t)=f_2(v(l,t))-f_2(\bar v(l,t))+\\\qquad g_2(v_t(l,t))-g_2(\bar v_t(l,t))\overset{\Delta}{=}\Lambda\tilde f+\Lambda\tilde g.\end{cases}\tag{18}$$

将方程(16)中的第一个方程与w_t取内积，第二个方程与$\bar w_t$取内积，然后两式相加得

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|w_t\|^2+\alpha\|w_{xx}\|^2+a\|w_x\|^2+\gamma\|\bar w_t\|^2+\\&\quad\delta\|\bar w_{xx}\|^2+\beta_0\|\bar w_x\|^2+4c(w_t,\bar w_t))+\\&\quad\eta_1\|w_t\|^2+\eta_2\|\bar w_t\|^2\\&=b(\|u_x\|^2-\|\bar u_x\|^2)(\bar u_{xx},w_t)+b\|u_x\|^2(w_{xx},w_t)-\\&\quad\alpha w_{xxx}(l)w_t(l)-\delta\bar w_{xxx}(l)\bar w_t(l)\end{aligned}\tag{19}$$

由边界条件(18)、中值定理与估计1、3对式(19)右端近似估计，最终得到某个常数$C>0$，

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(\|w_t\|^2+\alpha\|w_{xx}\|^2+a\|w_x\|^2+\gamma\|\bar w_t\|^2+\\&\quad\delta\|\bar w_{xx}\|^2+\beta_0\|\bar w_x\|^2+4c(w_t,\bar w_t))\\&\leqslant C(\|w_x\|^2+\|\bar w_x\|^2+\|w_t\|^2+\|w_{xx}\|^2).\end{aligned}$$

根据Growall引理，得$w=0,\bar w=0$，所以

$$u(x,t)=\bar u(x,t),\ v(x,t)=\bar v(x,t).$$

唯一性证毕.

因为$u_{xx},u_{xxt},v_{xx},v_{xxt}\in L^2(0,\infty;\ L^2(0,l))$，则$u(x,t),v(x,t)\in C^0([0,\infty);\ V)$，也可得

$u(x,t),v(x,t)\in C^1([0,\infty);\ U)$. 定理3.1证毕.

下面根据稠密性理论，证明系统(1)在初边值(2)–(3)下弱解的存在唯一性.

定理2.2　设空间H_0与函数$f_i(\cdot),g_i(\cdot)$的假设条件均成立，对于任何初值$(u^0,u^1,v^0,v^1)\in H_0$，系统(1)–(3)在H_0中存在唯一只与初值有关的弱解.

证明：　设$(u^0,u^1,v^0,v^1)\in H_0$，因为H_1在H_0中稠，则存在$(u_n^0,u_n^1,v_n^0,v_n^1)\in H_1$，使得

$u_n^0\to u^0$在V中，$u_n^1\to u^1$在U中.

$v_n^0\to v^0$在V中，$v_n^1\to v^1$在U中.

对于任意一个$n\in N$，存在(u_n,v_n)满足系统(1)–(3)，有下列式成立

$$\begin{cases} u_{ttn} + cv_{ttn} + \eta_1 u_{tn} + \alpha u_{xxxxn} - (a + b\|u_{xn}\|^2) u_{xxn} = 0, \\ cu_{ttn} + \gamma v_{ttn} + \eta_2 v_{tn} + \delta v_{xxxxn} - \beta_0 v_{xxn} = 0. \end{cases} \tag{20}$$

$$\begin{cases} u_n(x,0) = u_n^0(x), \quad u_{tn}(x,0) = u_n^1(x), \\ v_n(x,0) = v_n^0(x), \quad v_{tn}(x,0) = v_n^1(x). \end{cases} \tag{21}$$

$$\begin{cases} u_n(0,t) = u_{xn}(l,t) = u_{xxn}(0,t) = 0, \\ u_{xxxn}(l,t) = f_1(u_n(l,t)) + g_1(u_{tn}(l,t)), \\ v(0,t) = v_{xn}(l,t) = v_{xxn}(0,t) = 0, \\ v_{xxxn}(l,t) = f_2(v_n(l,t)) + g_2(v_{tn}(l,t)). \end{cases} \tag{22}$$

对方程(20)中的第一个方程与u_{tn}做内积,第二个方程与v_{tn}做内积,两式相加可得$\|u_{tn}\|^2 + \|v_{tn}\|^2 + \|u_{xxn}\|^2 + \|v_{xxn}\|^2 + \|u_{xn}\|^2 + \|v_{xn}\|^2 \leqslant C_0$. 其中$C_0$是一个与$n \in N$无关的正常数.设$(u_n, v_n)$,$(u_m, v_m)$是方程(20)的两组解,令$Z_{n,m} = u_n - u_m, \tilde{Z}_{n,m} = v_n - v_m, n, m \in N$,与正则解的唯一性步骤相似,并且考虑到$u_n^0, u_n^1, v_n^0, v_n^1$的收敛性,则存在$(u,v)$满足

$u_n \to u$ 在$C([0,T]; V)$强收敛,

$u_{tn} \to u_t$ 在$C([0,T]; U)$强收敛,

$v_n \to v$ 在$C([0,T]; V)$强收敛,

$v_{tn} \to v_t$ 在$C([0,T]; U)$强收敛.

由上面的收敛性,并取$n \to \infty$,则有

$$\begin{cases} u_{tt} + cv_{tt} + \eta_1 u_t + \alpha u_{xxxx} - (a + b\|u_x\|^2) u_{xx} = 0, \\ cu_{tt} + \gamma v_{tt} + \eta_2 v_t + \delta v_{xxxx} - \beta_0 v_{xx} = 0. \end{cases}$$

定理2.2证毕.

注:由定理2.2可知系统(1)-(3)能在空间H_0上定义一个连续半群.如果$(u^0, u^1, v^0, v^1) \in H_0$,那么可以引入映射

$\{S(t)\}_{t \geqslant 0}:(u^0, u^1, v^0, v^1) \to (u(t), u_t(t), v(t), v_t(t))$

它是H_0到H_0的一个映射并且满足半群性质,则$S(t)$是定义在H_0上的连续半群.

3 全局吸引子

定义3.1 对任何一个有界集$B \subset H$,存在$t_B = t(B) \geqslant 0$满足$S(t)B \subset \mathcal{B}, \forall t \geqslant t_B$,则有界集$\mathcal{B} \subset H$是半群$S(t)$的一个吸收集,定义$(H, S(t))$是一个耗散的动力系统[8].

定理3.2 在定理2.2的假设$(u_0, u_1, v_0, v_1) \in H_0$下,系统(1)-(3)相对应的半群$S(t)$在空间$H_0$上存在有界吸收集.

证明: 任取一个有界集$B \in H_0$,使初值$(u^0, u^1, v^0, v^1) \in B$,满足

$$(u(t), u_t(t), v(t), v_t(t)) = S(t)(u^0, u^1, v^0, v^1).$$

首先给出能量等式

$$E(t) = \frac{1}{2}(\|u_t\|^2 + \alpha\|u_{xx}\|^2 + a\|u_x\|^2 + \frac{b}{2}\|u_x\|^4 + \gamma\|v_t\|^2 + \delta\|v_{xx}\|^2 + \beta_0\|v_x\|^2) + \alpha\hat{f}_1(u(l,t)) + \delta\hat{f}_2(v(l,t)) + c(u_t, v_t). \tag{23}$$

定义辅助函数

$$\varphi(t) = \int_0^l (u_t u + \gamma v_t v + c v_t u + c u_t v + \frac{\eta_1}{2} u^2 + \frac{\eta_2}{2} v^2) \mathrm{d}x \tag{24}$$

系统(1)的第一个方程与$u_t + \varepsilon u$做内积,第二个方程与$v_t + \varepsilon v$做内积,然后将所得两式相加并整理有

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}E(t) + \varepsilon\frac{\mathrm{d}}{\mathrm{d}t}\varphi(t) + 2\varepsilon E(t) + \frac{b\varepsilon}{2}\|u_x\|^4 + \eta_1\|u_t\|^2 + \\ &\eta_2\|v_t\|^2 + \alpha g_1(u_t(l,t))u_t(l,t) + \delta g_2(v_t(l,t))v_t(l,t) \\ &= [2\varepsilon\alpha\hat{f}_1(u(l,t)) - \varepsilon\alpha f_1(u(l,t))u(l,t)] + \\ &[2\varepsilon\delta\hat{f}_2(v(l,t)) - \varepsilon\delta f_2(v(l,t))v(l,t)] - \\ &\varepsilon\alpha g_1(u_t(l,t))u(l,t) - \varepsilon\delta g_2(v_t(l,t))v(l,t) + \\ &4\varepsilon c(u_t, v_t) + 2\varepsilon\|u_t\|^2 + 2\gamma\varepsilon\|v_t\|^2 \end{aligned} \tag{25}$$

根据条件(5)得

$$|2\varepsilon\alpha\hat{f}_1(u(l,t)) - \varepsilon\alpha f_1(u(l,t))u(l,t)| \leqslant 0,$$
$$|2\varepsilon\delta\hat{f}_2(v(l,t)) - \varepsilon\delta f_2(v(l,t))v(l,t)| \leqslant 0.$$

根据条件(8)、条件(14),$\|u_x\|^2 \leqslant C_0$,C_0是一个只与B有关的正常数,结合Young不等式有

$$\begin{aligned} &|\varepsilon\alpha g_1(u_t(l,t))u(l,t)| \leqslant |3m_1\varepsilon\alpha u_t(l)u(l)| \\ &\leqslant \frac{3m_1\varepsilon\alpha}{2}|u_t(l)|^2 + \frac{3m_1\varepsilon\alpha l C_0}{2}, \\ &|\varepsilon\delta g_2(v_t(l,t))v(l,t)| \leqslant \frac{3m_2\varepsilon\delta}{2}|v_t(l)|^2 + \frac{3m_2\varepsilon\delta l C_0}{2} \end{aligned} \tag{26}$$

根据条件(7)得

$$|\alpha g_1(u_t(l,t))u_t(l,t)| \geqslant \alpha p_1|u_t(l,t)|^2,$$
$$|\delta g_2(v_t(l,t))v_t(l,t)| \geqslant \delta p_2|v_t(l,t)|^2.$$

把上述估计代入式(25),进一步整理得

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}E(t) + \varepsilon\frac{\mathrm{d}}{\mathrm{d}t}\varphi(t) + 2\varepsilon E(t) + \frac{b\varepsilon}{2}\|u_x\|^4 + \\ &(\eta_1 - 2\varepsilon c - 2\varepsilon)\|u_t\|^2 + (\eta_2 - 2\varepsilon c - 2\gamma\varepsilon)\|v_t\|^2 + \\ &(\alpha p_1 - \frac{3m_1\varepsilon\alpha}{2})|u_t(l,t)|^2 + (\delta p_2 - \frac{3m_2\varepsilon\delta}{2})|v_t(l,t)|^2 \\ &\leqslant \frac{3m_1\varepsilon\alpha l C_0}{2} + \frac{3m_2\varepsilon\delta l C_0}{2}, \end{aligned}$$

取 $0 < \varepsilon \leqslant \min\{2p_1/3m_1, 2p_2/3m_2, \eta_1/(2+c)\ \eta_2/(2c+2\gamma)\} = \varepsilon_0$足够小时,得到

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t) + \varepsilon\frac{\mathrm{d}}{\mathrm{d}t}\varphi(t) + 2\varepsilon E(t) \leqslant \frac{3m_1\varepsilon\alpha l C_0}{2} + \frac{3m_2\varepsilon\delta l C_0}{2}. \tag{27}$$

由能量等式(23),得

$$E(t) \geqslant \frac{\alpha}{2}\|u_{xx}\|^2 + \frac{\delta}{2}\|v_{xx}\|^2 + (1-\frac{c}{2})\|u_t\|^2 + (\gamma - \frac{c}{2})\|v_t\|^2 \geqslant \rho_0\|(u, u_t, v, v_t)\|^2_{H_0}. \tag{28}$$

其中 $\rho_0 = \min\{\alpha/2, \delta/2, (1-c)/2, (\gamma - c)/2\}$.

设 $E_\varepsilon(t) = E(t) + \varepsilon\varphi(t)$,则根据 $\varphi(t)$ 的定义并利用Young不等式,得

$$|E_\varepsilon(t) - E(t)| = |\varepsilon\varphi(t)| \leqslant \varepsilon C_1 E(t) \tag{29}$$

其中 $C_1 = \max\{1+2c,\ \gamma+2c, 1+c+\eta, \gamma+c+\eta_2\}$,不难推出对 ε 足够小时,有

$$\frac{1}{2}E(t) \leqslant E_\varepsilon(t) \leqslant \frac{3}{2}E(t) \tag{30}$$

结合 $E_\varepsilon(t)$ 的定义,同时把式(30)代入式(27)得

$$\frac{d}{dt}E_\varepsilon(t) + \frac{4\varepsilon}{3}E_\varepsilon(t) \leqslant \varepsilon(\frac{3m_1\alpha lC_0}{2} + \frac{3m_2\delta lC_0}{2}),$$

应用Growall不等式有

$$E_\varepsilon(t) \leqslant E_\varepsilon(0)e^{-\frac{4\varepsilon}{3}t} + [\frac{9}{8}C_0 l(m_1\alpha + m_2\delta)](1 - e^{-\frac{4\varepsilon}{3}t}) \tag{31}$$

因为正不变集 B 有界,$E_\varepsilon(0)$ 也有界,则存在 $t_B > 0$,当 t_B 足够大时,有

$$\frac{1}{2}E(t) \leqslant E_\varepsilon(t) \leqslant \frac{9}{8}C_0 l(m_1\alpha + m_2\delta) \tag{32}$$

从式(28)可得,$\forall t > t_B$,

$$\rho_0\|(u, u_t, v, v_t)\|^2_{H_0} \leqslant E(t) \leqslant \frac{9}{4}C_0 l(m_1\alpha + m_2\delta).$$

因此

$$\mathcal{B} = \{(u, u_t, v, v_t) \in H_0: \|(u, u_t, v, v_t)\|^2_{H_0} \leqslant \frac{9}{4\rho_0}C_0 l(m_1\alpha + m_2\delta)\}.$$ 是系统的一个有界吸收集.

引理3.3[9] 设 H 是一个巴拿赫空间,对于任何正不变有界集 $B \subset H, \forall\varepsilon > 0, \exists T = T(\varepsilon, B)$,满足 $\|S(T)x - S(T)y\| \leqslant \varepsilon + \phi_T(x, y), \forall x, y \in B$

这里 $\phi_T: H \times H \to R$ 满足对于任意序列 $\{z_n\} \in B$,

$$\liminf_{k\to\infty}\liminf_{l\to\infty}\phi_T(z_k, z_l) = 0$$

那么半群 $S(t)$ 在 H 中渐近光滑.

引理3.4[10] 若 $S(t)$ 是定义在度量空间 H 上的耗散的连续半群,当且仅当它在 H 中渐近光滑,则 $S(t)$ 在 H 中有紧的全局吸引子.

定理3.5 在定理2.2的假设 $(u^0, u^1, v^0, v^1) \in H_0$ 下,系统(1)-(3)相对应的半群 $S(t)$ 在 H_0 中渐近光滑.

证明: 任取一个有界集 $B \in H_0$,给定初值 $(u^0, u^1, v^0, v^1), (\bar{u}^0, \bar{v}^1, \bar{v}^0, \bar{v}^1) \in B$,设 $(u, v), (\bar{u}, \bar{v})$ 是系统的解,那么 $w = u - \bar{u}, \bar{w} = v - \bar{v}$ 满足系统(16)及边界条件(18).设能量等式

$$F(t) = \frac{1}{2}(\|w_t\|^2 + \alpha\|w_{xx}\|^2 + a\|w_x\|^2 + \gamma\|\bar{w}_t\|^2 + \delta\|\bar{w}_{xx}\|^2 + \beta_0\|\bar{w}_x\|^2 + b\|w_x\|^2\|u_x\|^2) + c(w_t, \bar{w}_t).$$

定义辅助函数

$$\psi(t) = \int_0^l (w_t w + \gamma\bar{w}_t\bar{w} + c\bar{w}_t w + cw_t\bar{w} + \frac{\eta_1}{2}w^2 + \frac{\eta_2}{2}\bar{w}^2)dx.$$

式(16)的第一个方程与 $w_t + \mu w$ 做内积,第二个方程与 $\bar{w}_t + \mu\bar{w}$ 做内积,将所得两式相加得

$$\begin{aligned}&\frac{d}{dt}F(t) + \mu\frac{d}{dt}\psi(t) + 2\mu F(t) + \eta_1\|w_t\|^2 + \eta_2\|\bar{w}_t\|^2 + \alpha\Lambda g w_t(l,t) + \delta\Lambda\tilde{g}\bar{w}_t(l,t)\\ &= b(\|u_x\|^2 - \|\bar{u}_x\|^2)(\bar{u}_{xx}, w_t) + b(\|u_x\|^2 - \|\bar{u}_x\|^2)(\bar{u}_{xx}, w) - \mu\alpha(\Lambda f + \Lambda g)w(l,t) - \mu\delta(\Lambda\tilde{f} + \Lambda\tilde{g})\bar{w}(l,t) - \alpha\Lambda f w_t(l,t) - \delta\Lambda\tilde{f}\bar{w}_t(l,t) +\\ &4\mu c(w_t, \bar{w}_t) + 2\mu\|w_t\|^2 + 2\gamma\mu\|\bar{w}_t\|^2 + \frac{b}{2}\|w_x\|^2\frac{d}{dt}\|u_x\|^2.\end{aligned} \tag{33}$$

由 $\|u_x\|, \|\bar{u}_x\|, \|\bar{u}_{xx}\| \leqslant C_0$ 与Young不等式,得

$$\begin{aligned}&b(\|u_x\|^2 - \|\bar{u}_x\|^2)(\bar{u}_{xx}, w_t)\\ &= b(u_x + \bar{u}_x, u_x - \bar{u}_x)(\bar{u}_{xx}, w_t)\\ &\leqslant b\|u_x + \bar{u}_x\| \cdot \|u_x - \bar{u}_x\| \cdot \|\bar{u}_{xx}\| \cdot \|w_t\|\\ &\leqslant \frac{bC_0}{2}\|w_x\|^2 + \frac{bC_0}{2}\|w_t\|^2\end{aligned} \tag{34}$$

$$b(\|u_x\|^2 - \|\bar{u}_x\|^2)(\bar{u}_{xx}, w) \leqslant \frac{bC_0}{2}\|w_x\|^2 + \frac{bC_0}{2}\|w\|^2 \tag{35}$$

根据条件(6)得

$$\begin{aligned}&|\mu\alpha\Lambda f w(l,t)| \leqslant \mu\alpha 3k_1|w(l)|^2 \leqslant 3\mu\alpha k_1\|w\|^2_\infty \leqslant 3\mu\alpha k_1 l\|w_x\|^2\\ &|\mu\delta\Lambda\tilde{f}\bar{w}(l,t)| \leqslant \mu\delta 3k_2|\bar{w}(l)|^2 \leqslant 3\mu\delta k_2\|\bar{w}\|^2_\infty \leqslant 3\mu\delta k_2 l\|\bar{w}_x\|^2\end{aligned} \tag{36}$$

根据条件(8)得

$$\begin{aligned}|\mu\alpha\Lambda g w(l)| &\leqslant \mu\alpha 3m_1|w_t(l)| \cdot |w(l)|\\ &\leqslant \mu\alpha 3m_1|w_t(l)|\sqrt{l}\|w_x\|\\ &\leqslant \frac{3m_1\mu\alpha\sqrt{l}}{2}|w_t(l)|^2 + \frac{3m_1\mu\alpha\sqrt{l}}{2}\|w_x\|^2,\\ |\mu\delta\Lambda\tilde{g}\bar{w}(l)| &\leqslant \mu\delta 3m_2|\bar{w}_t(l)||\bar{w}(l)|\\ &\leqslant \frac{3m_2\mu\delta\sqrt{l}}{2}|\bar{w}_t(l)|^2 + \frac{3m_2\mu\delta\sqrt{l}}{2}\|\bar{w}_x\|^2\end{aligned} \tag{37}$$

根据条件(6)得

$$|\alpha\Lambda f w_t(l,t)| \leqslant 3\alpha k_1 |w(l)|\cdot|w_t(l)| \leqslant 3\sqrt{l}\,\alpha k_1\|w_x\|\cdot|w_t(l)| \leqslant \frac{3\sqrt{l}\,\alpha k_1}{2}\|w_x\|^2+\frac{3\sqrt{l}\,\alpha k_1}{2}|w_t(l)|^2,$$

$$|\delta\Lambda\tilde{f}\bar{w}_t(l,t)| \leqslant 3\delta k_2|\bar{w}(l)|\cdot|\bar{w}_t(l)| \leqslant \frac{3\sqrt{l}\,\delta k_2}{2}\|\bar{w}_x\|^2+\frac{3\sqrt{l}\,\delta k_2}{2}|\bar{w}_t(l)|^2 \tag{38}$$

根据条件(7)得

$$\begin{cases}\alpha\Lambda g w_t(l,t) \geqslant \alpha p_1|w_t(l,t)|^2,\\ \delta\Lambda\tilde{g}\bar{w}_t(l,t) \geqslant \delta p_2|\bar{w}_t(l,t)|^2.\end{cases} \tag{39}$$

利用有界性得

$$\left|\frac{b}{2}\|w_x\|^2\frac{\mathrm{d}}{\mathrm{d}t}\|u_x\|^2\right| = |b\|w_x\|^2\int_0^l u_x u_{xt}\,\mathrm{d}x| \leqslant b\|w_x\|^2\|u_{xx}\|\cdot\|u_t\| \leqslant bC_0\|w_x\|^2. \tag{40}$$

将上述估计(34)–(40)代到式(33),整理得到

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}F(t)+\mu\frac{\mathrm{d}}{\mathrm{d}t}\psi(t)+2\mu F(t)+\eta_1\|w_t\|^2+\eta_2\|\bar{w}_t\|^2+\alpha p_1|w_t(l)|^2+\delta p_2|\bar{w}_t(l)|^2\\ &\leqslant (2bC_0+3\mu\alpha k_1 l+\frac{3m_1\mu\alpha\sqrt{l}}{2}+\frac{3\sqrt{l}\,\alpha k_1}{2})\|w_x\|^2+(3\mu\delta k_2 l+\frac{3m_2\mu\delta\sqrt{l}}{2}+\frac{3\sqrt{l}\,\delta k_2}{2})\|\bar{w}_x\|^2+\\ &(\frac{3m_1\mu\alpha\sqrt{l}}{2}+\frac{3\sqrt{l}\,\alpha k_1}{2})|w_t(l)|^2+\frac{bC_0}{2}\|w\|^2+(\frac{3m_2\mu\delta\sqrt{l}}{2}+\frac{3\sqrt{l}\,\delta k_2}{2})|\bar{w}_t(l)|^2+\\ &(\frac{bC_0}{2}+2\mu c+2\mu)\|w_t\|^2+(2\mu c+2\gamma\mu)\|\bar{w}_t\|^2\end{aligned} \tag{41}$$

当μ充分小时,有

$$\begin{cases}\eta_1-(\frac{bC_0}{2}+2\mu c+2\mu)\geqslant 0,\\ \eta_2-(2\mu c+2\gamma\mu)\geqslant 0,\\ \alpha p_1-(\frac{3m_1\mu\alpha\sqrt{l}}{2}+\frac{3\sqrt{l}\,\alpha k_1}{2})\geqslant 0,\\ \delta p_2-(\frac{3m_2\mu\delta\sqrt{l}}{2}+\frac{3\sqrt{l}\,\delta k_2}{2})\geqslant 0.\end{cases}$$

所以,当$0<\mu\leqslant \min\{\frac{2\eta_1-bc}{4c+4},\frac{\eta_2}{2c+2\gamma},\frac{2p_1-3\sqrt{l}\,k_1}{3\sqrt{l}\,m_1},\frac{2p_2-3\sqrt{l}\,k_2}{3\sqrt{l}\,m_2}\}$足够小时,有

$$\frac{\mathrm{d}}{\mathrm{d}t}F(t)+\mu\frac{\mathrm{d}}{\mathrm{d}t}\psi(t)+2\mu F(t)\leqslant C_2(\|w_x\|^2+\|\bar{w}_x\|^2+\|w\|^2) \tag{42}$$

其中

$$C_2=\max\{3\mu\alpha k_1 l+\frac{3m_1\mu\alpha\sqrt{l}}{2}+\frac{3\sqrt{l}\,\alpha k_1}{2}+2bC_0,\ 3\mu\delta k_2 l+\frac{3m_2\mu\delta\sqrt{l}}{2}+\frac{3\sqrt{l}\,\delta k_2}{2}\}.$$

定义$F_\mu(t)=F(t)+\mu\psi(t)$,通过类似证吸收集时的讨论知当μ足够小时,有

$$\frac{1}{2}F(t)\leqslant F_\mu(t)\leqslant\frac{3}{2}F(t) \tag{43}$$

由式(42)、式(43)得

$$\frac{\mathrm{d}}{\mathrm{d}t}F_\mu(t)+\frac{4\mu}{3}F_\mu(t)\leqslant C_2(\|w_x\|^2+\|\bar{w}_x\|^2+\|w\|^2),$$

$t\geqslant 0$.上式利用Growall引理,得

$$F_\mu(t)\leqslant F_\mu(0)e^{-\frac{4}{3}\mu t}+C_2\int_0^t e^{-\frac{4}{3}\mu(t-s)}(\|w_x(s)\|^2+\|\bar{w}_x(s)\|^2+\|w(s)\|^2)\mathrm{d}s.$$

另一方面有

$$F_\mu(t)\geqslant\frac{1}{2}F(t)\geqslant\frac{1}{4}\rho_1(\|w_{xx}\|^2+\|w_t\|^2+\|\bar{w}_{xx}\|^2+\|\bar{w}_t\|^2).$$

其中$\rho_1=\min\{1-c,\alpha,\gamma-c,\delta\}$.因此能找到一个只依赖于$B$的常数$C_B$,使得

$$\|w(t),w_t(t),\bar{w}(t),\bar{w}_t(t)\|_{H_0}\leqslant\frac{2}{\sqrt{\rho_1}}C_B e^{-\frac{2}{3}\mu t}+\frac{2\sqrt{C_2}}{\sqrt{\rho_1}}(\int_0^T(\|w_x(s)\|^2+\|\bar{w}_x(s)\|^2+\|w(s)\|^2)\mathrm{d}s)^{\frac{1}{2}}. \tag{44}$$

对于任意给定的$\varepsilon>0$,存在T,当T足够大时有

$$\frac{2}{\sqrt{\rho_1}}C_B e^{-\frac{2}{3}\mu T}<\varepsilon. \tag{45}$$

并且定义$\phi_T: H_0\times H_0\to R$为

$$\phi_T((u^0,u^1,v^0,v^1),(\bar{u}^0,\bar{u}^1,\bar{v}^0,\bar{v}^1))=\frac{2\sqrt{C_2}}{\sqrt{\rho_1}}\times(\int_0^T(\|u_x(t)-v_x(t)\|^2+\|\bar{u}_x(t)-\bar{v}_x(t)\|^2+\|u(t)-v(t)\|^2)\mathrm{d}s)^{\frac{1}{2}}.$$

然后从式(44)–式(45),可得对任意的$(u^0,u^1,v^0,v^1),(\bar{u}^0,\bar{u}^1,\bar{v}^0,\bar{v}^1)\in B$,有

$$\|S(T)(u^0,u^1,v^0,v^1)-S(T)(\bar{u}^0,\bar{u}^1,\bar{v}^0,\bar{v}^1)\|_{H_0}\leqslant\varepsilon+\phi_T((u^0,u^1,v^0,v^1),(\bar{u}^0,\bar{u}^1,\bar{v}^0,\bar{v}^1)).$$

都成立.

令$(u_n^0,u_n^1,v_n^0,v_n^1)$是集合$B$中给定的序列,因为$B$是有界的,且是正不变的,所以系统的解的对应序列$(u_n(t),u_{tn}(t),v_n(t),v_{tn}(t))$在$H_0$中一致有界.因此$\{u_n\},\{v_n\}$在$C^0([0,\infty);V)\cap C^1([0,\infty);U)$中有界.根据嵌入定理$V\mapsto H_0^1$是紧的,则存在子序列

$\{u_{nk}\}$, $\{v_{nk}\}$ 在 $C([0,\infty);\ H_0^1(\Omega))$ 中一致强收敛.

$$\lim_{k\to\infty}\lim_{l\to\infty}\int_0^T \|u_{xnk}(s)-u_{xnl}(s)\|^2+\|v_{xnk}(s)-v_{xnl}(s)\|^2 ds$$

$$+\lim_{k\to\infty}\lim_{l\to\infty}\int_0^T \|u_{nk}(s)-u_{nl}(s)\|^2 ds=0,$$

则有

$$\lim_{k\to\infty}\lim_{l\to\infty}\phi_T((u_{nk}^0,u_{nk}^1,v_{nk}^0,v_{nk}^1),(u_{nl}^0,u_{nl}^1,v_{nl}^0,v_{nl}^1))=0.$$

因此,半群 $S(t)$ 在空间 H_0 中渐近光滑.

定理 3.6　根据引理 3.4,定理 3.2,3.5 得系统所确定的半群 $S(t)$ 在空间 H_0 中有全局吸引子.

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ATTRACTOR FOR EQUATIONS OF ABENDING AND TORSIONAL COUPLED BEAM UNDER NONLINEAR BOUNDARY CONDITIONS *

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Abstract　In this paper, attractor of beam equations is studied, which is excited jointly by winding and twisting with a realistic background under nonlinear boundary conditions. Firstly, we prove the existence and uniqueness of global solution by the Faedo-Galerkin method. Secondly, we prove that the system has bounded absorbing sets and asymptotic smoothness of the related solution semigroup. Finally we get the existence of the global attractor.

Key words　coupled flexural and torsional,　nonlinear boundary conditions,　global solution,　global attractor

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